A Stackelberg Game Framework for Mobile Data Gathering in Leasing Residential Sensor Networks

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Abstract—This paper studies a data gathering problem in a wireless sensor network containing multiple private residual sub-networks. The interaction between the wireless sensor network operator and the owners of residual sub-networks is modeled by a Stackelberg game, which forms a novel framework for jointly analyzing the pricing, gathering data, and planning routes. It is shown that the game has a unique Stackelberg equilibrium at which the wireless sensor network operator sets prices to minimize total cost, while owners of residual sub-networks respond accordingly to maximize their utilities subject to their bandwidth constraints. An algorithm and theoretical analyses are provided for the corresponding strategies of the operator and owners, and validated by extensive simulations. It is demonstrated that the algorithm achieves lower network cost compared with existing data gathering strategies.

Index Terms—Wireless sensor networks, Stackelberg game, equilibrium, mobile data gathering, distributed algorithms, convex optimization.

I. INTRODUCTION AND RELATED WORK

Analysis of big data plays an important role in the decision makings of large companies, governments and even in the daily life of individuals. Consistent collections of all information from various kinds of sensors existing around us contribute to a large portion of data. Sensors collecting information such as temperature, humidity, solar irradiance, motions, stretch, toxins, etc. form networks to realize the efficient, timely, and perpetual data collection for some areas.

In recent years, many studies about the Wireless Sensor Networks (WSNs) have been conducted. In [1], a WSN based on the static and mobile energy harvesting from the RF signals transmitted by the readers extends lifetime greatly. In [2], some heuristic algorithms are proposed to derive the routes of data collection vehicles while jointly considering the energy-efficiency, delay-awareness and lifetime-balance of the sensor networks. In [3], the abundance of deployed sensors is utilized to increase the lifetime and reduce the cost of the WSN while ensuring the k-coverage of targets at any time.

Many realistic sensor networks have been built to collect data. For example, in the field of viticulture, sensor networks in the vineyard manipulating the temperature, humidity, maturity and other indices provide important data to guide the cultivation [4]. And also, sensor networks consisting of solar irradiance sensors collect precisely temporal and spatial solar data globally for many years, which acts as the most important data in the construction of solar plants, household sunroofs and other applications about solar irradiance [5].

It is observed that due to the low cost of wireless sensors and fluctuations of data flows, the sensors are usually deployed at high density, so there exists abundant bandwidth and other resources during the normal operations of the sensor network. Meanwhile, the data collected from the sensor networks can be applied in many different applications simultaneously. For example, the collected solar irradiance data can be applied in the estimation of harvested solar power or can be applied in the weather forecast [6]. These abundant utilities can be leased to other consumers of data, which allows the owners of WSNs to make an extra profit while freeing other data consumers from constructing another network in the same position. In [8], the authors consider a mobile communication network in which the private WiFi resources are leased to serve the requirements of mobile users.

Motivated by these features of WSNs and the potential win-win situations between owners and consumers, we introduce the game theory to study the equilibrium of the game between both sides [7]. The basic concept of homo economicus in the game theory assumes that the participants of a game are economically rational, which infers that owners of networks are willing to lease spare resources if they can make profits and consumers are willing to rent resources if the total costs can be reduced. The data from different sensor networks can be collected by a Mobile Data Collector (hence called SenCar) roaming around the field at the pre-set output port of any network.

In this paper, we consider a wireless sensor network which contains different residential sensor networks (RSNs). For wireless sensor network operator (WSNO), we study the problem of optimizing the data gathering cost and providing incentives to owners of RSNs. WSNO gathers data from a certain geographic area, which is divided into two different types. In Type I Area, WSNO collects data from the sensors deployed by itself. In Type II Area, WSNO offers incentives to the owners of RSNs, and accordingly, owners of RSNs lease the spare utilities to provide data for WSNO.

Deciding the optimal incentives offered to RSNs owners is a challenging economic problem for the reason that WSNO needs to consider the different properties of RSNs. To address the problem, the interaction between WSNO and owners of RSNs is modeled as a dynamic Stackelberg game. The leader of the game, WSNO, determines the pricing policy. The owners of RSNs are the followers and they only need to respond with the available amount of bandwidth they are willing to offer.

We formulate the joint incentives, data gathering and routing problem which considers the interaction between the WSNO and RSNs owners as a Stackelberg game. The problem is complicated, it is divided into to subproblems and a primal-dual method is applied to solve one of them. The main contributions of this paper are as followings.

A Novel Framework of Wireless Sensor Network. We introduce a new framework containing multiple game participants, WSNO and private RSNs, and also jointly analyzing the pricing, data gathering and routing problem encountered.

Stackelberg Game Model. By modeling the interaction be-
between WSNO and RSNs owners as a dynamic Stackelberg game, we state the equilibrium of the game, solve the problem in two stages, and present the best strategy for WSNO and RSNs owners.

Performance Evaluations. We present a theoretical analysis and extensive simulation results to validate the convergence of the proposed algorithm and demonstrate that our algorithm can achieve lower network cost than the compared data gathering strategy.

The remainder of the paper is organized as follows. Section II introduces the system model, and provides the formulation of the cost minimization problem by analyzing the strategies of WSNO and RSNs owners in a two-stage Stackelberg game. Section III decomposes the problem into two subproblems, derives a theorem to solve the first subproblem, and a primal-dual algorithm to solve the second subproblem. Section IV presents the simulation results of the proposed algorithm. Finally, Section V concludes the study.

II. SYSTEM MODEL AND PROBLEM FORMULATION

In this section, we consider a WSN which contains several small RSNs. These RSNs belong to separately private owners or parties. The WSNO, which manages the sensors, motivates owners of private RSNs to collect data. We then formulate the joint incentive, routing and data collecting problem to minimize the total cost for collecting fixed amount of data from the WSN. As shown in Fig.1, the network consists of two parts: Type I Area and Type II Area.

A. Type I Area Model

In Type I Area, a WSN which consists of a set of static sensors, denoted by $\mathcal{N}$, is considered. In this area, the sensors are deployed by the WSNO. WSNO needs to determine the proper positions to place sensors and collect data from sensors. We study the anchor-based gathering scheme where the mobile collector gathers data directly from sensors by visiting each anchor point in a periodic data gathering tour. Here, anchors are the locations for collecting data packets. We denote the set of anchors as $\mathcal{A}$.

In this framework, the SenCar is used to collect data from anchors by approaching an anchor at a time. SenCar is customized from an off-the-shelf battery-powered vehicle. The battery exhausting time of the SenCar is denoted by $T$. The SenCar is equipped with computational components, radio transceiver and location device, so it can be commanded remotely from the base station by WSNO to perform different tasks in the sensing field. When the SenCar moves to an anchor $a$ ($a \in \mathcal{A}$), it stays near the anchor for a period of sojourn time (denoted as $t^a$), serving as the WSNO which sends the transmitting data requests to each sensor and schedule these sensors to upload sensing data. Important notations used in this paper are summarized in Table I.

Since SenCar could gather the data from sensors (including the sensors located at the anchors), which could be linked within one hop from the anchors, sensors in one-hop coverage of anchor point $a$ is denoted as set $\mathcal{N}^a$, and the number of sensors in $\mathcal{N}^a$ is denoted as $N^a$. We assume that each sensor has enough cache to buffer data. When SenCar stops at anchor $a$, sensor $i$ ($i \in \mathcal{N}^a$) uploads amount of data $x_i^a$. To ensure the WSNO gathers enough data from sensors, a minimum data amount $X^a$ is set for anchor $a$ which is equally as the minimum amount of data that the sensors covered by the anchor transmits to the SenCar. For simplicity, assume that the sets of sensors covered by different anchors are disjoint, i.e.,

$$\sum_{i \in \mathcal{N}^a} x_i^a \geq X^a. \quad (1)$$

The amount of data is also constrained by the bandwidth of sensors and staying time for the SenCar stopping at anchors of the sensor network. The data transmission band from the sensor to SenCar is fixed as $B$ bytes, which leads the following constraint

$$\sum_{i \in \mathcal{N}^a} \frac{x_i^a}{p_i^a} \leq B \cdot t^a. \quad (2)$$

The process of data uploading is not ideally lossless due to the unreliable channels in wireless sensor networks. [9], the successful data transition rate from sensor $i$ to the SenCar is denoted as $p_i^a$. The parameter $p_i^a$ describes the channel condition of the sensor $i$. It is affected by the several other factors such as the location of the sensor, transition environment, and channel delay.

The process of collecting data from sensors incurs a certain cost for satisfying gather amount of data due to the consumption of its network resources, such as monetary cost, energy consumption or other metrics modeling user application needs. The cost of data collection depends on the amount of data which the WSNO gets. We introduce a general cost function $C^a(\cdot)$ with the requirement that a strictly convex, increasing and twice-differentiable function with respect to the amount of data that sensor $i$ uploads to the SenCar at anchor point $a$ (i.e., $x_i^a$). According to our model, the total cost for collecting data is the sum of all sensors from all anchor points.

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<th>Table I. Important Notation</th>
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B. Type II Area Model

In the Type II Area, a set of $R$ regions, denoted by $\mathcal{R}$ are covered by the RSNs which belong to the separate and private parties or owners, thus regions covered by these RSNs are non-overlapping. The set of RSNs is denoted as $\mathcal{M}$. In each RSN, there is one data center which is linked with all the sensors of RSN. The data center is the caching machine to store all the data sent from these sensors by transmission links directly.

The RSNs are managed by private owners who are considered rational and self-interested entities. For the RSN ($m \in \mathcal{M}$), the total maximum available bandwidth from sensors to the data center is denoted as $B^m > 0$. We denote $V^m(b)$ as the utility function perceived by RSN owner $m$ for using $b$ bytes bandwidth of sensors to transmit data for serving its own data collection needs.
needs. Suppose that these utility functions are increasing and strictly concave functions related to the amount of consumed resource. The assumption of concavity is reasonable since when the bandwidth needs increase for the RSNs owners, the increasing rate of the utility goes down. The RSNs owners lease the bandwidth of links to WSN, which transmit data from sensors to data centers. WSN can collect data directly from the data center of each RSN. To achieve this, WSN set a data transmission machine (DTM) for every data center. DTM performs the task of transmitting data to the SenCar in the wireless channel like the sensors in Type I Area. However, it has more capacity to store data and larger bandwidth to transmit a large amount of data.

For the reason that RSNs belong to private owners, some data in the specified time period are private which may not be accessible to others. The visiting of SenCar may also interrupt the daily working schedule of the RSNs. Therefore SenCar is not allowed to stop at DTM for collecting data without the permission of RSNs’ owners. To solve this problem, the most available time for RSN \( m \) to stop for collecting data is denoted as \( t^m (t^m \geq 0) \).

Each RSN \( m \in M \) leases total \( b^m \) bytes of bandwidth to the WSN and uses remaining bandwidth to satisfy its own needs. The leasing incentive offered by WSN is \( s^m \). Hence, utility function of RSN \( m \) can be written:
\[
U^m (b^m, s^m) = V^m (B^m - b^m) + b^m (s^m - \rho^m),
\]
where \( \rho^m \) is the statistic incurred serving cost to transmit data for WSN.

To ensure that the WSN gets enough data from the area of RSNs, the minimum amount of data collected from the area covered by the RSN in the region \( r \in R \) is denoted as \( X^r \). There are two cases that WSN prefers to collect data by placing sensors itself instead of renting the bandwidth of RSNs. One case is that the bandwidth rented from the RSNs owners could not satisfy the minimum demand for collecting data. The other case is that cost for renting bandwidth exceed the cost incurred during collecting data from the sensors placed by WSN. The set of wireless sensor networks built by the WSN is denoted as \( M \). In the region \( r \), the set of small WSN is denoted as \( m \in M \), the number of sensors is denoted as \( N^m \). Similar to the Type I Area, we select a set of anchor points in wireless sensor network \( m \) which denoted as \( A^m \). The set of sensors covered by anchor \( a^m \in A^m \) is modeled as \( N^a^m \); the number of sensors in set is \( N^a^m \). SenCar would stop at the anchor points \( a^m \) for a sojourn time \( t^a^m \) to collect data. The amount of data collected from sensor \( k \in N^a^m \) is \( x_k^a^m \).

The minimum amount of data collected from region \( r \) should satisfy the following the constraint:
\[
x^m + \sum_{a^m \in A^m} \sum_{k \in N^a^m} x_k^a^m \geq X^r, \forall r \in \mathcal{R}
\]

For the RSN owners, SenCar collects data from DTM of each RSN owner, the amount of data is restricted by the total leasing bandwidth from RSNs and the sojourn time,
\[
x^m / \rho^m \leq b^m \cdot t^m.
\]

In (5), successful data transmission rate from sensor \( k \) to SenCar is \( p^m \). Similarly, for the data collected from sensors built in the area of RSN by WSN, Successful data transmission rate from sensor \( k \) to SenCar is \( p^m \). The constraint is below,
\[
\sum_{k \in N^a^m} x_k^a^m \leq B \cdot t^a^m.
\]

### C. Joint incentive, routing and data collecting problem

The WSN can lease bandwidth capacity from the owners of RSNs, the interaction among the WSN and RSNs owners is modeled as a two-stage hierarchical Stackelberg game [12]. We state the game as follow: The players are composed of two parts, WSN and the set of RSNs owners. WSN is the leader, accordingly, the set of RSNs owners are followers. In Stage I, WSN decides the incentives \( s^m \), the sojourn time for the SenCar at a specific anchor and how much data they should collect from each sensor. In Stage II, RSNs owners determine the amount of the bandwidth of every sensor they are willing to lease as the response to the incentives given by the WSN. The payoff of the WSN is the serving cost and for the residential sensor network owner is the utility function. We denote the leasing cost function for WSN to collect data from the area of the private sensor network owner as \( H(\cdot) \). Summarizing, the cost function \( J(\cdot) \) of WSN is the sum of the cost incurs by collecting data and the cost for leasing bandwidth resources of the RSNs owners:
\[
J(\cdot) = \sum_{a \in A} \sum_{i \in N^a} C_i^a (x_i^a) + H(\cdot).
\]
The solution of this game is a specific case of Stackelberg game called equilibrium. The outcome \{\( b^m, s^m \)\} of this two stage Stackelberg game reaches the equilibrium if the following conditions are satisfied at the same time:
\[
U^m (b^m, s^m) \leq U^m (b^{m*}, s^{m*}), \quad \forall b^m, s^m
\]
\[
J(x_1^a, b^{m*}, s^{m*}, x_m^m, x_k^a^m, t^a, t^m) \leq J(x_1^a, s^m, b^m, x_m^m, x_k^a^m, t^a, t^m), \quad \forall b^m, s^m
\]
The case in Stage II is firstly addressed, where the optimal strategy is considered for the RSN owner \( m \) to make decisions about allocation of bandwidth by solving an optimization problem. This problem takes the incentives offered by WSN as input. More specifically, the allocation problem, we name it \( RSN_m \), for each RSN \( m \) is defined as:
\[
\begin{align*}
RSN_m : & \max_{s^m \in \mathcal{S}} U^m (b^m, s^m), \\
\text{s.t.} & 0 \leq b^m \leq B^m
\end{align*}
\]
where, Eq. (10) represents the bandwidth constraints.

The solution of each RSN \( m \) depends on the function \( V^m (\cdot) \). Also, \( V^m (\cdot) \) is a strictly concave and monotonous function with a convex compact set. Following the principle of diminishing returns, we assume the user utility to be logarithmically related to the portion of consumed resources which has been widely used in literature is,
\[
V^m (b) = \beta^m \log(b),
\]
where \( \beta^m \geq 0 \) is a scaling parameter that shows the different RSNs owners value the bandwidth of RSN \( m \).

Using Eq. (11), the solution of the allocation problem is:
\[
b^m (s^m) = B^m - \beta^m \frac{s^m}{s^m - \rho^m}.
\]
The equation is limited by two constraints, i.e. \( s^m \geq \rho^m \) and \( b^m \leq B^m \).
In stage I, the WSN0 determines the reimbursement prices (incentives) offered to the RNs’ owners and stopping time at each anchor and data collecting policy. In doing so, it considers the anticipated strategy of each private sensor network owners in stage II.

Clearly, the stopping time at each anchor is constrained by the battery exhausting time of the SenCar. Hence, the following condition should hold:

\[ \sum_{m \in M} t^m + \sum_{a \in A} t^a + \sum_{a \in A} t^m_a \leq T, \quad (13) \]

Therefore, the leasing function of the WSN0 could be denoted as:

\[ H(s^m, b^m, x^m, x^m_k, t^m) = \sum_{m \in M} C^m(x^m) + \]

\[ \sum_{m \in M} b^m \cdot s^m + \sum_{m \in M} \sum_{a \in A} \sum_{m \in N^a} C^m_k(x^m_k), \quad (14) \]

\[ C^m_k(\cdot) \] denotes the cost function for WSN0 collecting data from sensor \( k \) built by itself covered in anchor \( a^m \in A^m \). Similar to the function \( C^m(\cdot) \), these two functions are strictly convex, increasing and twice-differentiable function with the amount of data they collect.

Hence, we can formulate the Joint Incentive, Routing, Data collecting problem (JIRD, for short) that minimizes the WSN0’s total cost for collecting data and providing incentives:

\[ J(x^a_t, s^m, b^m, x^m, x^m_k, t^m_a, t^m) = \sum_{a \in A} \sum_{x^m} C^m_t(x^m_t) + H(s^m, b^m, x^m, x^m_k, t^m), \quad (15) \]

s.t. constraints:

1. \( t^m_a \geq 0, \forall m \in M \) \quad (16)
2. \( t^a \geq 0, \forall a \in A \) \quad (17)
3. \( s^m \geq \rho^m, \forall m \in M \) \quad (18)
4. \( x^a_i \geq 0, \forall i \in N, a \in A, k \in N^a \) \quad (19)
5. \( x^m \geq 0, \forall m \in M \) \quad (20)
6. \( x^m_k \geq 0, \forall m \in M, a \in A, k \in N^a \) \quad (21)
7. \( 0 \leq b^m \leq B^m, \forall m \in M \) \quad (22)

III. PROBLEM DECOMPOSITION AND SOLUTION

In this section, the JIRD problem is decomposed into two subproblems. We establish the optimal condition for the first subproblem and propose a solution to solve the second based on the primal-dual method.

A. JIRD Decomposition

According to the system model mentioned earlier, the area of the WSN is divided into two parts, Type I Area and Type II Area. Accordingly, we separate the optimization problem JIRD into two subproblems, and these two subproblems are named as \( P_I \) and \( P_{II} \) respectively, as follows,

\[ P_I : \min \sum_{a \in A} \sum_{x^m} C^m_t(x^m_t), \quad (23) \]

s.t. (1), (2), (13), (17), (19),

and

\[ P_{II} : \min \ H(s^m, b^m, x^m, x^m_k, t^m). \quad (24) \]

**Table I: Algorithm 1: Primal-Dual Algorithm**

| Input: \( X^a, T^a, B^m, s^m, b^m, t^m \) |
| \( C^m(\cdot), C^m(\cdot), C^m(\cdot) \) |
| Outputs: \( b^m, s^m, t^m \) |

Initially, set dual variables \( \lambda^1 \) and \( \mu^1 \) to 0.0003, the lower bound as \( LB = -\infty \), the upper bound as \( UB = +\infty \), \( l = 1 \).

while true

1. Solve the subproblem (in parallel).
   - Solve subproblem \( P_{I}^{(1)} \), finding \( x^m \) and \( s^m \);
   - Solve subproblem \( P_{II}^{(2)} \), finding \( t^m \) and \( t^m \);
   - Name \( Q(\lambda^{(l)}), \mu^{(l)} \) the optimal value of the primal problem;
   - if \( Q(\lambda^{(l)}), \mu^{(l)} > LB \) then \( LB = Q(\lambda^{(l)}, \mu^{(l)}); \)

end

Set UB as the optimal value of \( P_{II}^{(1)}; \)
Update UB;

2. Update dual variables;
   - Using (32)(33);
   - if \( \frac{UB-UP}{UP} < 0.01 \) and \( l < 50 \) then \( l = l + 1 \);
break

Notice that both subproblems have the constraint Eq. (13) in which \( t^a \) and \( t^m \) coexist. The following theorem shows the optimal solution of \( P_{II} \). After getting the optimal solution, \( t^a \) and \( t^m \) in Eq. (13) can be decoupled.

**Theorem 1.** The optimal solution of the \( P_{II} \), non-negative matrices \( x = \{ x^a_i \}_{a \in A} \) holds if and only if

\[ \sum_{i \in N^a} x^a_i = X^a. \quad (25) \]

The proof of this theorem is omitted due to space limitations, and will be provided in an expanded version of this paper.

To give more feasibility to solve \( P_{II} \), the optimal solution of \( P_I \), the non-negative vector \( t = \{ t^m \}_{a \in A} \) satisfies the following constraint,

\[ \sum_{i \in N^a} x^a_i = B \cdot t^m. \quad (26) \]

According to Theorem 1, Eq. (13) is equivalent to,

\[ \sum_{m \in M} t^m_a + \sum_{a \in A} x^m_a \leq T, \quad (27) \]

where \( \hat{T} = T - \sum_{a \in A} x^a \). With fixed \( t^m_a \), the constraint Eq. (13) is successfully decoupled.

B. Solution to \( P_{II} \)

In order to solve the \( P_{II} \) in its general form, we appeal to the primal-dual method [11]. For the constraint in Eq. (27), we introduce Lagrangian multiplier \( \lambda \), and for the constraint in Eq. (4), we introduce the set of Lagrangian multiplier,

\[ \mu = (\mu^r : \forall r \in R). \quad (28) \]

Then, the Lagrange function is defined as follows.

\[ L(s^m, b^m, x^m, x^m_k, t^m, \mu, \lambda) = H(s^m, b^m, x^m, x^m_k, t^m) + \]

\[ \sum_{r \in R} \mu^r(x^m - x^m - \sum_{a \in A} \sum_{m \in N^a} x^m_a) + \]

\[ \lambda \left( \sum_{m \in M} t^m + \sum_{a \in A} t^m_a - \hat{T} \right), \quad (29) \]
After introducing the Lagrangian multiplier $\lambda$ and $\mu$, the problem can be rewritten as,

$$\max_{\mu', \lambda} \min_{s^m, b^m, x^m, x^a_k, t^m} L(s^m, b^m, x^m, x^a_k, t^m, \mu, \lambda)$$

$$s.t. (4), (5), (6), (12), (13), (16), (18), (20), (21), (22),$$

$$\lambda \geq 0,$$  \hspace{1cm} (30)

$$\mu' \geq 0, \forall r \in R.$$  \hspace{1cm} (31)

The index of subgradient iterations is denoted as $l$. In each iteration, the dual variables are updated, and the primal problem is solved in order to update the primal variable. Moreover, the primal variables, in turn, update the dual variables. The details are described below and summarized in the Algorithm 1.

The Dual Problem. A subgradient method [10] is adopted which converges to the optimal problem. In each iteration of $l = 1, 2, 3, \ldots$, for the dual problem, the following formula gives the updated dual variables [10],

$$\lambda^{(l+1)} = [\lambda^{(l)} + \delta^{(l)} \nabla L' (\lambda)]^+, \hspace{1cm} (32)$$

$$\mu^{(l+1)} [\mu^{(l)} + \delta^{(l)} \nabla L' (\mu^{(l)})]^+, \hspace{1cm} (33)$$

where $\delta^{(l)}$ is the step size for the $l$ update. $\nabla L' (\lambda)$ is the gradient of the dual variable $\lambda$ and $\nabla L' (\mu^{(l)})$ is the gradient of the dual variable of $\mu^{(l)}$.

The Primal Problem. The primal problem is strictly convex, while the dual is differentiable. The primal problem is solved after the update of the dual variables in each iteration $l$. The primal problem can be decomposed into two different classes of problems denoted as $P_{II}^{(1)}$ and $P_{II}^{(2)}$ respectively, which are as follows,

$$P_{II}^{(1)} : \min \sum_{m \in \mathcal{M}} C^m (x^m) + \sum_{m \in \mathcal{M}} b^m \cdot s^m \cdot t^m +$$

$$\lambda \sum_{m \in \mathcal{M}} t^m - \sum_{r \in \mathcal{R}} \mu^r x^m,$$  \hspace{1cm} (34)

$$s.t. (5), (12), (18), (20), (22), (30), (31),$$

and,

$$P_{II}^{(2)} : \min \sum_{a^m \in \mathcal{A}^m} \sum_{k \in \mathcal{N}^m} C^a_k (x^a_k) + \lambda \sum_{a^m \in \mathcal{A}^m} t^m$$

$$- \lambda \mu^r \sum_{r \in \mathcal{R}} \sum_{a^m \in \mathcal{A}^m} x^a_k - \sum_{r \in \mathcal{R}} \mu^r \cdot X^r,$$  \hspace{1cm} (35)

$$s.t. (6), (16), (21), (30), (31).$$

Leasing Subproblem. Note that $P_{II}^{(1)}$ only contains the variables about the RSN $m \in \mathcal{M}$, such as the leasing bandwidth variable $b^m$, the incentive price for leasing bandwidth $s^m$ and $x^m$ amount of data collected from RSN $m$, which is named Leasing Subproblem here. $P_{II}^{(1)}$ could further be decomposed into $M$ independent sub-subproblems, which is denoted as $P_{II}^{(1)(m)}$. Notice that its objective is strictly convex and the constraint sets are also convex. Therefore, the standard convex optimization techniques can be applied [11].

WSNO Subproblem. Similarly, $P_{II}^{(2)}$ only involves procedures that WSNO collects data from sensors deployed by itself, which is named WSNO Subproblem. The objective function is strictly convex and the constraint sets are also convex. Hence, it can be solved by using standard convex optimization techniques [11].

IV. PERFORMANCE EVALUATIONS

In this section, we conduct extensive simulations to demonstrate the usage and efficiency of the proposed algorithm. We simulate the following schemes:

1) Baseline: WSNO gathers data from sensors deployed by itself and does not rent bandwidth of the RSNs.

2) Primal-dual: WSNO can rent bandwidth of RSNs to collect data. The problem is solved by using the primal-dual algorithm.

Simulation Setup. We consider that one WSNO collects data in a specific area. Unless specified, in Type I Area, the number of sensors is 20 and the number of anchor points is 4. The minimum data demand for each anchor $a$ is $X^a = 800$ bytes. The wireless data transmission channel bandwidth from each sensor to SenCar is 250 bytes. The data successful transmission rate from sensors deployed by WSNO $p^m$ ranges from 0.7 to 1. In Type II Area, the number of RSN is 1, the number of sensors deployed by WSNO is 12 and among these sensors, the number of anchor points is 3. The maximum available bandwidth of RSN is 500 bytes. The successful data transmission rate of RSN $p^m$ ranges from 0.8 to 1. The successful data transmission rate from sensors deployed by WSNO $p^m_a = p^m$. The minimum amount of data collected from the Type II Area is 5000 bytes. The battery exhausting time $T$ for the SenCar is 80 seconds, and the most available sojourn time to stop at RSN $t^m$ is 20 seconds.

The performance criteria that we consider is the WSNO serving cost, as shown in Eq.(15). The cost function to collect data from RSN $m \in \mathcal{M}$ is defined as $C^m (x^m) = \omega^m (x^m)^2$ for each $m \in \mathcal{M}$, where $\omega^m$ is the weight of cost function for SenCar to gather data from RSN $m$. Similarly, we define the cost function to collect data from sensor $i$ covered by anchor $a$ as $C^i_a (x^i_a) = \omega^i_a (x^i_a)^2$ and the cost function to collect data from sensor $k$ covered by anchor $a^m$ as $C^a_k (x^a_k) = \omega^a_k (x^a_k)^2$. We set weight parameters of sensors deployed by WSNO as $\omega^i_a$ and $\omega^a_k$, which are in the same range, from 0.1 to 0.8, and the weight parameter of RSN is 0.2.

Convergence of Primal-dual Algorithm. We illustrate the convergence of the Primal-dual algorithm in this part. The step size of the algorithm is set as $1/ (1 + 2\beta)$. Fig. 2 shows the evolution of network cost in Type II Area, Lagrange multiplier $\mu$, sojourn time variable $t^m$, and data variable $x^m$, $x^a_k$ versus the number of iterations in the Primal-dual algorithm. Lower bound (LB) is set as the value of the Eq.(29) and upper bound (UB) is set as the value of Eq. (14). The optimal solution would be reached if $UB - LB$ is small enough. It can be seen from Fig.2a that cost of Type II Area first increases sharply in the first few iterations because of the setting of the initial value of the Lagrange multiplier $\mu$ and $\lambda$. The cost curve drops fast in a few iterations and then slightly decreases until it reaches the optimum. Fig.2g shows the convergence of sojourn time at different anchor points, and it can be seen that they all approach to the stable state fast. It further validates that at any iteration step, the sojourn time is feasible to the primal problem. Fig.2f shows the leased bandwidth from RSN, and the result also converges. It can be concluded that Primal-dual algorithm could reach the optimal solution and the equilibrium of Stackelberg game is also reached. Comparing the amount of data from RSN and the data from sensors deployed by WSNO, WSNO prefers...
collecting more data from RSN, because it costs less to rent bandwidth for collecting data from RSN than collecting data from sensors deployed by WSNO. This demonstrates that the leasing framework of WSNs is effective and feasible to reduce the cost for gathering data.

![Figure 2: Evolution of network cost](image)

**Figure 2:** Evolution of network cost (a) The convergence of Primal-dual algorithm. (b) The amount of data collected from RSN. (c) Amount of data from sensors covered by anchor 1. (d) Amount of data from sensors covered by anchor 2. (e) Amount of data from sensors covered by anchor 3. (f) Amount of bandwidth leased from RSN. (g) Sojourn time from different anchors. (h) Network cost comparison between primal-dual algorithm and baseline based on minimum amount of data in Type II Area.

**Network cost in Type II Area.** In this part, we conduct a suite of simulations to evaluate the network cost achieved by the Primal-dual algorithm for \( P_{11} \). To meet the lowest requirement for simulations, in Type II Area, we consider 25 sensors deployed by WSNO and 5 anchor points among these sensors, where each anchor covers five sensors. The rest of simulation settings are the same as what mentioned above.

Fig.2h plots the network costs of the Primal-dual algorithm and network costs without leasing bandwidth from RSN when the minimum demand \( x^r \) varies from 1000 bytes to 5000 bytes. It is shown that the network cost increases with \( X^r \). This can be explained by the intuition that collecting more data results in more cost. The performance of the Primal-dual algorithm is better than the framework without leasing RSN. It can save the monetary cost to lease RSNs to collect data when compared with the traditional framework which does not have leasing.

**V. Conclusions**

In this paper, we have studied the performance optimization for a novel framework of the wireless sensor network where a set of residual sensor networks coexist. We formalized the problem as a joint incentives, data gathering and routing problem. We characterized this problem as a two-stage dynamic Stackelberg game and decomposed it into two subproblems and described a primal-dual algorithm to solve the second subproblem. The minimum network cost can be achieved when reaching the equilibrium of the Stackelberg game. Using extensive numerical results, we identified the convergence of the algorithm and the key parameters that affect the performance of the framework. Moreover, by comparing its performance with non-leasing RSN framework, we showed that this framework can significantly reduce the cost for gathering data in WSN.

**REFERENCES**


