An Efficient Online Market Mechanism for Resource Leasing in Cloud Radio Access Networks

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***Abstract*—This work studies the emerging C-RAN market in a 5G wireless network where mobile operators lease computation and communication resources from the tower company to serve wireless users. We propose an online C-RAN auction where each mobile operator bids for three types of resources in a future time window: wireless spectrum at base stations (BSs), front-haul link capacities, and mobile BS instances at the mobile cloud. We target an online C-RAN auction that executes in polynomial time, elicits truthful bids from mobile operators, and maximizes the social welfare of the C-RAN eco-system with both spectrum cost at BSs and server cost at the mobile cloud considered. We show how the marriage of (i) a new Fenchel dual approach to convex optimization with (ii) the posted pricing framework for online auction design can help achieve the three goals simultaneously, and evaluate the efficiency of our online C-RAN auction through both theoretical analysis and empirical studies.**

I. Introduction

As new wireless devices and services proliferate, the demand for high-throughput and high-efficiency wireless communication has been ever escalating. In response, the 5G cellular standard is emerging for wireless networks in the near future [1], promising a $1,000\times$ improvement in peak data rates (over LTE 2010) and an ultra-low end-to-end latency within 5 ms. The key technologies for enhancing system capacity and transmission efficiency in 5G networks include massive MIMO transmission over millimeter waves, cloud computing based radio access networks (C-RAN), mobile service infrastructure separation, network function virtualization (NFV), software defined networking (SDN) and network coding.

A particular infrastructure revolution in 5G is the deployment of C-RAN, migrating functionalities including signal processing and optimized control decision making from the radio access networks into the cloud. The signal processing for A/D and D/A conversion in the traditional radio access network happens at cell site. Capacity of the signal processor in each base station (BS) is customarily over-provisioned to accommodate peak communication demands. The utilization per BS is relatively low as traffic volume fluctuates dramatically in the temporal domain, with average network load trailing far behind peak load. As a result, the computing resource in a BS is wasted as it cannot be shared with other BSs. In the recent past, C-RAN [2] has emerged as a promising solution to this problem. C-RAN decouples the remote radio heads (RRH) from the signal processing units. The new, streamlined BS is equipped with simplified RRHs. It communicates with user equipments, and transmits sampled analog signals to the mobile cloud via a front-haul network. As a canonical instance of NFV, the mobile cloud hosts a pool of virtual BS instances to realize signal processing. Since 2011, a number of mobile operators started to migrate towards the C-RAN paradigm with pilot deployments. For example, China Mobile recently cooperated with ZTE to deploy a trial C-RAN system in Changesha [3]. Korea Telecom constructed the world's first commercial Cloud-RAN, Cloud Communications Center (CCC), which can manage 144 BSs per server and accommodate 1,000 servers in each data center [4]. Centralized information processing in the C-RAN infrastructure enables mobile operators to scale their network provisioning in accordance with demand, and further facilitates system-wide optimization and cost-saving.

Another revolution in 5G is the separation of the cellular infrastructure from mobile service provisioning. Mobile network operators in today's market face increasing pressure as the price competition continues to ramp up. Meanwhile, the cost to build, operate and upgrade BSs continues arising, and constitutes a major portion of the operation cost. For example, from 2008-2012, the expense for operators in China to maintain and construct BSs is around $\$50$ billion per year, but the utilization rate of the radio access network is only $1/3$ [5]. The infrastructure cannot be shared among operators for the centralized optimization. The telecom industry realized the problem, and the "tower" company was incarnated. The tower company is responsible for designing, constructing, and maintaining BSs, then leases BSs to operators. In the US, Crown Castle took over AT&T's BS subsystem in October 2013, and became the largest provider of shared wireless infrastructure in the US with approximately $40,000$ towers [6]. American Tower runs the tower business around the world with approximately 96,000 BSs [7]. In China, a national Tower Company was founded in July 2014, inheriting BSs from the three major mobile operators [8]. In the C-RAN paradigm, the tower company is dedicated to maintain system infrastructure, providing cost-effective infrastructure solutions. Mobile operators can focus on their core business and lease resources (spectrum resources at BSs, virtual BS instances and

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front-haul bandwidth) from the tower company to realize their virtual radio access network.

In the 5G C-RAN market, both long term and short term leases are natural options for mobile operators. The long term contract is to satisfy their baseline service needs, while short term leases may cover their temporal and spatial demand spikes. In this work, we propose an online auction mechanism for the short-term C-RAN resource leasing. A main alternative to the auction approach would be a fixed pricing scheme. While simple and intuitive, fixed pricing comes with its limitations. First, what price to charge is a tricky question and the tower company may end up with trial-and-error. Second, upon fluctuations in the demand-supply relation, fixed pricing risks over-pricing and under-pricing that lead to missed revenue opportunities. In comparison, the auction approach automatically discovers the right market price and adapts it dynamically with supply-demand. A well designed auction can further elicit truthful bids and allocate C-RAN resources to operators who value them the most.

Our work is among the first on online C-RAN resource sharing, and it targets the following properties. i) *Truthfulness:* Bidding true valuation is a dominant strategy for each operator. ii) *Computational efficiency:* The algorithms for winner determination and payment calculation execute in polynomial time. iii) *Social welfare maximization:* We aim to maximize the aggregate utility of the tower company and mobile operators with operation cost of the tower company considered. In practice, upon the arrival of each bid, the tower company must immediately decide whether to accept it or not, but the optimal offline allocation requires complete knowledge of the market over the entire lifespan. Even in the offline setting, the resource sharing problem has a packing structure that is NP-hard. The challenge further escalates when operation cost at BSs and the mobile cloud are factored in. Most existing work in wireless spectrum auctions [9] and virtual machine auctions [10] fail to consider the operation cost of the service provider. Service cost indeed exists in practice and plays an important role in computing the seller's utility and hence social welfare.

We formulate the social welfare maximization problem in the C-RAN auction into an online convex optimization problem. We resort to the primal-dual optimization framework, and adopt a new application of Fenchel duality for deriving the dual of the convex problem. The structure of the dual problem guides the design and the analysis of the online auction, handling the convex cost function that represents the operation cost of the C-RAN system.

We then design a primal-dual online auction to solve the social welfare maximization problem with a good competitive ratio in polynomial time, which can handle the departures of resource requests, such that resources released by leaving bids can be reused by later bids. The winner determination process of our mechanism depends only on the current unit price of the resources and bidders' demand. The key technique lies in the design of the unit price function which is updated according to the operation cost of the tower company and the usage level of resources. Thus, our mechanism falls into the family of *posted pricing mechanism* [11], which guarantees truthfulness by making sure that the winner determination process is bid independent. In our mechanism, a bidder cannot obtain a higher utility by lying about its biding price; as a result, submitting a truthful bid constitutes a dominant strategy for each bidder.

We proceed to discuss another business model of the tower company and evaluate our auction through a large scaled simulation study. It is possible that the tower company focuses on the maintenance of base stations and the front-haul of C-RAN, but leases cloud service from other cloud service provider. We show that our primal-dual framework can handle the extended model as well. Simulation studies reflect that our mechanism can always achieve a close-to-optimal social welfare under practical settings. The competitive ratio remains at a low level (< 2) and is mostly much better than the worst case bound proven by theoretical analysis.

In the rest of the paper, Sec. II reviews related work. We define the system model and introduce the background of the truthful auction design in Sec.III. Sec. IV presents our online auction design and Sec. V evaluates the performance of our auction. Sec. VI concludes the paper.

II. Related Work

The application of the auction approach to resource allocation in different computing and communication systems has been an active area of research. A series of recent work focuses on auction mechanism design in cloud computing systems. Zaman *et al.* propose combinatorial auction-based allocation mechanisms for virtual machine instances [12]. Zhang *et al.* [10] study dynamic resource allocation in cloud computing. The above literature considers an implicit offline setting. Although Shi *et al.* [13] and Zhang *et al.* [14] investigate online auctions, they only focus on resource allocation in the cloud and ignore servers' operation cost. In addition, Shi *et al.* [13] assume each acquired VM is used for only one round and Zhang *et al.* [14] simplify the model such that the cloud service provider only provides one type of cloud resource. Zhang *et al.* [15] study the online cloud auction and consider the cost of the cloud server. Our work has a different design in the price function. We further consider the role of fiber links and BSs, and their operation cost.

A plethora of studies exists on the auction design for the secondary spectrum markets. Gandhi *et al.* focus on conflict-free spectrum allocations to maximize the revenue [16]. Wang *et al.* [17] propose a truthful online double auction for spectrum allocation, relying on a limiting assumption that users' requests follow Poisson distribution. The spectrum auction literature often focuses on wireless interference and spectrum reuse, while this work addresses challenges posed by an online packing optimization problem, including non-linear server costs.

Along the direction of C-RAN architecture and operations, Zhu *et al.* introduce the first prototype of a virtual BS pool for WiMAX on a multi-core IT platform [18]. Liu *et al.* [19] reconfigure the architecture of the front-haul in C-RAN.

Recently, researchers start to study the resource allocation problem in C-RAN. Pompili *et al.* [20] introduce co-location models for provisioning and allocation of virtual BSs and propose different virtual BS architectures. Morcos *et al.* [21] and Parsaeefard *et al.* [22] propose algorithms for C-RAN resource allocation, but with no proven performance guarantee. Zhu *et al.* [23] propose an offline auction mechanism to jointly address the hierarchical resource allocation problem in 5G networks. In a real-world C-RAN market, operators' requests may not arrive simultaneously, and need to be handled immediately. Therefore, it is natural to consider the online scenario. Gu *et al.* [24] propose the first online C-RAN auction. In their model, although bidders arrive online, they only bid for current resources. Our work proposes the first online auction across the temporal domain that can allocate future resources with the consideration of operation cost.

The online primal dual method is a power algorithmic technique that has been utilized to solve broad problems (see [25] for a detailed survey). This work has been partly inspired by a recent work in theoretical computer science on non-linear primal-dual approximation algorithm design [11]. The optimization problem in our work has different structure, requiring new solution techniques.

III. System Model

In this section, we first introduce the system model in Sec. III-A and formulate the social welfare maximization problem in the C-RAN auction in Sec. III-B.

A. C-RAN Model and Auction Preliminaries

As shown in Fig. 1, we consider a cloud radio access network (C-RAN) where M base stations are connected to a cloud data center via optical fiber links (front-haul) [2]. The tower company owns the C-RAN infrastructure, and provides two types of resource leases to mobile network operators. Long term leases are subscribed by operators for meeting baseline data communication demands of their customers. The tower company further conducts an online auction to respond to operators' short-term demand hikes (*e.g.,* a shopping festival in a mall). During the online auction, an operator who faces a resource shortage submits a C-RAN resource bid to the cloud.

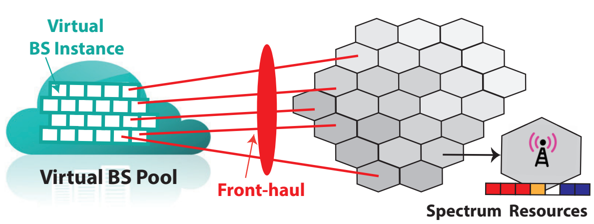

Fig. 1. An illustration of the C-RAN auction with three resources (spectrum resources at BSs, virtual BS instances in the cloud and front-haul link bandwidth).

Let $[X]$ denote the integer set $\{1, 2, \ldots, X\}$. During a long time period of T slots, a total number of N bids are submitted to the tower company. Let B_n be the nth bid that was submitted at time t_n, and let t_n^- and t_n^+ be its start and end times for occupying the requested resources, where $t_n \le t_n^- < t_n^+$. Multiple bids can arrive simultaneously, and would be ordered randomly. A unit spectrum resource is controlled by a corresponding virtual BS instance. Our auction model is also applicable to the scenarios where a virtual BS instance manages multiple spectrum units. Let $r_{n,m}(t)$ be the number of unit spectrum resources requested at BS m by bid n in slot t, then $\sum_{m\in[M]} r_{n,m}(t)$ represents the total number of virtual BS instances requested by bid n in time t. Here the value of $r_{n,m}(t)$ can remain the same or vary between t_n^- and t_n^+. We use R_m and C to denote the available amount of spectrum resources at BS m and the capacity of the virtual BS pool at the cloud, respectively. Bidders also contend for front-haul bandwidth to transmit baseband signal to the mobile cloud. Our model supports two front-hual topologies: point-to-point, *i.e.,* every BS has its own fiber to the cloud, and daisy chain, *i.e.,* BSs share fiber links to the cloud [26]. Assume there are K fiber links between BSs and the mobile cloud. Let $d_{n,k}(t)$ be the amount of bandwidth required at the kth fiber link in bidder n's bid at time t. The capacity of the kth fiber link is D_k. Let b_n be the bidding price of bid n to lease the three types of resources between t_n^- and t_n^+, and v_n be the true valuation of bid n. A bid can be expressed as follows:

$$B_n = \{t_n^-, t_n^+, \{r_{n,m}(t)\}_{m\in[M], t\in[t_n^-, t_n^+]}, \{d_{n,k}(t)\}_{k\in[K], t\in[t_n^-, t_n^+]}, b_n\}, \forall n \in [N].$$

After receiving bid n, the tower company immediately decides whether to accept it, and computes the payment if so. The binary variable x_n is 1 if bid n is accepted, and 0 otherwise. Let p_n be the payment of a winning bid n. The utility of bid n with bidding price b_n is: if $x_n = 1$, $u_n(b_n) = v_n - p_n$; otherwise, $u_n(b_n) = 0$. The utility of the tower company equals the aggregate user payments minus its operation cost.

The operation cost of a C-RAN system mainly consists of two parts: the cost of the virtual BS pool at the cloud, and the cost at base stations. In comparison, the operational cost at the front-haul is relatively minor and is ignored. For front-haul links, we focus on their capacity limits instead [26]. Our solution framework can be extended to handle front-haul cost if desired. The operation cost of the virtual BS pool is mainly due to power consumption for provisioning virtual BS instances, which increases as the number of running virtual BS instances grows. The cost at BSs mainly comes from power consumption of RRHs and the usage of wireless spectrums. The tower company purchases spectrum licences from governmental agencies (*e.g.*, the Federal Communications Commission in the United States) through spectrum auctions [27], and incurs substantial cost in doing so. The overall cost at BSs is approximately linear to the amount of allocated spectrum resources [2]. The cost function $f(y(t))$ takes the amount of currently allocated resources $y(t)$ as input, and is defined as following:

$$f(y(t)) = \begin{cases} \beta_1 y(t)^{1+\gamma} + \beta_2 y(t), & \text{if } y(t) \in [0, C] \\ +\infty, & \text{otherwise.} \end{cases} \quad (1)$$

$\beta_1 y(t)^{1+\gamma}$ is the cost at the virtual BS pool, where β_1 is a coefficient determined by electricity prices. $\gamma \ge 0$ modulates the shape of the function, following the specific operation model of physical servers in the mobile cloud. For instance, Dynamic Voltage Frequency Scaling (DVFS) is a technique to adjust the frequency or voltage of a CPU to control its power consumption [28]. When the increment of the voltage is proportional to the usage of CPU, power consumption exhibits a cubic increase [29]. In comparison, power consumption at the cloud is roughly linear to the utilization of the CPU, RAM and disk when DVFS is disabled [30]. The function of a virtual BS instance is realized by a virtual machine that is assembled from physical resources (CPU, RAM, disk), the value of γ can be approximately set from within $[0.5, 2.2]$ for the overall power consumption [30]. $\beta_2 y(t)$ is the cost at BSs, and is linear to the number of allocated spectrum units. Table I summarizes notation for easy reference.

TABLE I
SUMMARY OF NOTATIONS

N	# of bids	$[X]$	integer set $\{1,\dots,X\}$
M	# of base stations	K	# of fiber links
T	# of time slots	$t_n^-(t_n^+)$	start (end) time of bid n
f	cost function	f^*	convex conjugate of f
$r_{n,m}(t)$	# of spectrum resources at BS m in bid n at slot t		
$d_{n,k}(t)$	# of bandwidth at link k in bid n at slot t		
b_n	bidding price of bid n		
x_n	bid n wins (1) or not (0)		
R_m	amount of available spectrum resources at BS m		
C	capacity of virtual BS instances pool		
D_k	capacity of kth fiber link		
$s(t)$	marginal price per unit resource at t		
u_n	bid n's utility		
R_{max}	$\max_{n,m,t}\{R_m/r_{n,m}(t)\}$		
U	$\max_{n\in[N]}\{b_n/\sum_{m\in[M]}\sum_{t\in[t_n^-,t_n^+]} r_{n,m}(t)\}$		
p_n	payment of bid n in A_{online}		

In a future 5G network, it is also possible for the tower company to own only BSs and the front-haul, while renting VMs from a cloud service provider. We discuss this alternative scenario in Sec. IV-C.

The bidders are assumed to be selfish; they may choose to lie about their true valuation, if doing so may lead to a higher utility. The C-RAN ecosystem as a whole has a social "happiness" that is our target of optimization. Towards such optimization, it is important to elicit truthful bids from bidders.

Definition (Truthfulness in bidding price): A C-RAN auction is *truthful in bidding price* if for any bid n, reporting its true valuation is its dominant strategy, and this strategy always maximizes its utility. That is, for all $b_n \neq v_n$, $u_n(v_n) \geq u_n(b_n)$.

Definition (Social welfare): The social welfare in the C-RAN auction over time span T equals the tower company's utility ($\sum_{n\in[N]} p_n x_n - \sum_{t\in[T]} f(y(t))$) plus the bidders' aggregate utility ($\sum_{n\in[N]}(v_n - p_n)x_n$), *i.e.,* $\sum_{n\in[N]} v_n x_n - \sum_{t\in[T]} f(y(t))$ (payments cancel themselves).

B. Social Welfare Maximization in The C-RAN Auction

Under the assumption of truthful bidding, *i.e.,* $b_n = v_n$, the social welfare maximization problem in the C-RAN resource auction can be formulated into the following convex optimization problem:

$$\text{Maximize} \sum_{n\in[N]} b_n x_n - \sum_{t\in[T]} f(y(t)) \tag{2}$$

$$\text{Subject To:} \sum_{\substack{n\in[N]:\\ t_n^-\leq t\leq t_n^+}} r_{n,m}(t)x_n \leq R_m, \forall m\in[M], \forall t\in[T], \tag{2a}$$

$$\sum_{\substack{n\in[N]:\\ t_n^-\leq t\leq t_n^+}} \sum_{m\in[M]} r_{n,m}(t)x_n \leq y(t), \forall t\in[T], \tag{2b}$$

$$\sum_{\substack{n\in[N]:\\ t_n^-\leq t\leq t_n^+}} d_{n,k}(t)x_n \leq D_k, \forall k\in[K], \forall t\in[T], \tag{2c}$$

$$x_n\in\{0,1\}, y(t)\geq 0, \forall n\in[N], \forall t\in[T]. \tag{2d}$$

Constraint (2a) states that the amount of the aggregate spectrum resources of successful bids at each BS cannot exceed the BS's spectrum capacity. The amount of the total virtual BS instances at time t from the accepted bids is $y(t)$. By setting $f(y(t)) = +\infty$ when $y(t) > C$, constraint (2b) guarantees the allocation of the virtual BS instances will never violate the instance pool's capacity limit. Constraint (2c) guarantees that data transmission rate is limited by the bandwidth capacity at front-haul links. Binary decision making (to accept each bid or not) is enforced by constraint (2d).

If we relax the integral constraint of x_n to $x_n \geq 0$ and introduce dual variable $z_m(t)$, $s(t)$ and $w_k(t)$ to constraints $(2a)$, $(2b)$ and $(2c)$, the Fenchel dual [31] of the relaxed convex program (2) can be formulated as following.

$$\text{Minimize} \sum_{m\in[M]}\sum_{t\in[T]} R_m z_m(t) + \sum_{k\in[K]}\sum_{t\in[T]} D_k w_k(t) + \sum_{t\in[T]} f^*(s(t)) \tag{3}$$

Subject To:

$$\sum_{m\in[M]}\sum_{t\in[t_n^-,t_n^+]} (r_{n,m}(t)z_m(t) + r_{n,m}(t)s(t)) + \sum_{k\in[K]}\sum_{t\in[t_n^-,t_n^+]} d_{n,k}(t)w_k(t) \geq b_n, \forall n\in[N], \tag{3a}$$

$$z_m(t)\geq 0, s(t)\geq 0, \forall m\in[M], \forall t\in[T]. \tag{3b}$$

where $f^*(s(t))$ is the convex conjugate [32] of the cost function $f(\cdot)$, defined as:

$$f^*(s(t)) = \sup_{y(t)\geq 0}\{s(t)y(t) - f(y(t))\} \tag{4}$$

Proposition 1: The conjugate of cost function $f(y(t))$ is

$$f^*(s(t)) = \begin{cases} \left(\dfrac{s(t)-\beta_2}{1+\gamma}\right)^{\frac{1+\gamma}{\gamma}} \cdot \dfrac{\gamma}{\beta_1^{\frac{1}{\gamma}}}, & y_0(t)\leq C \\ Cs(t) - \beta_1 C^{1+\gamma} - \beta_2 C, & y_0(t) > C \end{cases} \tag{5}$$

where $y_0(t) = (\frac{s(t)-\beta_2}{\beta_1(1+\gamma)})^{\frac{1}{\gamma}}$.

Proof:

$$f^*(s(t)) = \sup \begin{cases} s(t)y(t) - \beta_1 y(t)^{1+\gamma} - \beta_2 y(t), & y(t)\in[0,C] \\ s(t)y(t) - \infty, & y(t) > C \end{cases}.$$

We only need to consider the conjugate of f when $y(t)\in[0,C]$, as $s(t)y(t) - \infty = -\infty$ when $y(t) > C$.

Let $g(y(t)) = s(t)y(t) - \beta_1 y(t)^{1+\gamma} - \beta_2 y(t)$. The derivative of $g(y(t))$ with respect to $y(t)$ can be expressed as:

$$g(y(t))' = s(t) - \beta_1(1+\gamma)y(t)^\gamma - \beta_2.$$

The local maximum happens at the point $y_0(t)$ when $g(y_0(t))' = 0$, whose solution is $y_0(t) = (\frac{s(t)-\beta_2}{\beta_1(1+\gamma)})^{\frac{1}{\gamma}}$.

Note that the value of $y(t)$ must lie in the range of $[0, C]$, thus the supremum of $g(y(t))$ is $y_0(t)$ only if $y_0(t) \in [0, C]$. Otherwise, when $y_0(t) > C$, we can obtain that $g(y(t))' > 0$, which indicates that $g(y(t))$ monotonically increases as the growth of $y(t)$ and the supremum happens at $y(t) = C$.

In conclusion, we derive the conjugate of the cost function as shown in (5). □

Problem (2) is a convex program with concave objective function and linear constraints. The optimal offline solution requires complete knowledge about the system over the entire time span T, which is impractical. Upon the arrival of each bid, the tower company must immediately decide whether to accept this bid. In the next section, we leverage the dual problem (3) to design an online primal-dual allocation algorithm, and tailor a payment scheme that works in concert with it.

IV. Online Auction Mechanism

We next design a truthful online auction that solves the C-RAN social welfare maximization problem (2) with good competitive ratio in polynomial time. The auction mechanism is presented in Sec. IV-A. Detailed analysis of its properties including truthfulness and competitiveness in social welfare are in Sec. IV-B. Sec. IV-C extends the discussions to an alternative business model of the tower company.

A. Online Auction Design

We consider the non-linear cost function with $\gamma > 0$, the case with $\gamma = 0$ will be discussed later in Sec. IV-C. The tower company determines whether to serve a bid n or not upon its arrival. If bid n is accepted, primal variable x_n is set to 1, and $y(t)$ is increased by $\sum_{m\in[M]} r_{n.m}(t)$ for $t \in [t_n^-, t_n^+]$. The cost of the tower company also grows accordingly to serve bid n. Otherwise, x_n is zero and $y(t)$ remains intact.

1) Resource Allocation: The key problem is how to decide the value of x_n. We seek help from the dual program (3), to update the value of primal variables. If we interpret the dual variables $s(t)$ as the marginal price per unit of resource at time t, then $\sum_{m\in[M]} \sum_{t\in[t_n^-, t_n^+]} r_{n,m}(t)s(t)$ is the total price of resources requested in bid n. The marginal price $s(t)$ is determined by the cost function (1) and will be discussed later. Note that according to the setting of the cost function, the front-haul doesn't generate any cost, therefore, bidders don't need to pay for it. Let u_n be the utility of bid n. Each bid's utility is a non-negative value:

$$u_n = \max\{0, b_n - \sum_{m\in[M]} \sum_{t\in[t_n^-, t_n^+]} r_{n,m}(t)s(t)\}.$$

Our online auction algorithm accepts bid n as long as its has a positive utility and the assignment of x_n is feasible for the primal problem (2). A winning bid n will be charged $\sum_{m\in[M]} \sum_{t\in[t_n^-, t_n^+]} r_{n,m}(t)s(t)$ for leasing resources from the C-RAN infrastructure.

For the feasibility of the dual problem, we update other dual variables as follows. Because dual variables $w_k(t)$ are associated to the front-haul constraint (2c) which doesn't influence the operation cost, the value of $w_k(t)$ is set to zero. Furthermore, from complementary slackness conditions, $x_n = 1$ only when the corresponding dual constraint becomes tight. Thus, we let $z_m(t) = u_n / \sum_{m\in[M]} \sum_{t\in[t_n^-, t_n^+]} r_{n,m}(t), \forall m \in [M], \forall t \in [t_n^-, t_n^+]$. Here the value of $z_m(t)$ can be interpreted as the unit utility of bid n at time t.

2) Marginal Price Design: There are two goals in the marginal price design. One is to ensure that the unit price is at least the unit operation cost, such that the tower company's revenue is non-negative. Another is to ensure that early bids with low values will not consume much capacity, such that latter bids with high value still have an opportunity to win.

In the offline solution, the marginal price $s(t)$ can be defined as the differential of the cost function, *i.e.,* $f'(\hat{y}(t))$ where $\hat{y}(t)$ is the overall demand of resources at time t if $\hat{y}(t) \le C$. Recall that a unit spectrum resource is controlled by a virtual BS instance and the front-haul incurs no operation cost. Thus $\hat{y}(t)$ is the final demand of virtual BS instances at time t, and is also equivalent to the final demand of spectrum resources. Moreover, the marginal price acts as the criteria to select high value bids if the final demand exceeds the capacity such that the amount of allocated resources is at most equal to the capacity.

However, the tower company doesn't have complete knowledge of the entire system in the online setting. It can only use the current demand to predict the final demand. Our approach is to assume that the final demand is δ ($\delta > 1$) times of the current demand $y(t)$ if the final demand is below the capacity. The marginal price under this scenario is set to $f'(\delta y(t))$. When the predicted final demand exceeds the capacity, the marginal price grows exponentially in the current demand, *i.e.,* $s(t) = f'(C)e^{\sigma(y(t)-\frac{C}{\delta})}$. The value of σ should be carefully designed such that the final amount of allocated virtual BS instances $\le C$. To sum up, the marginal price per unit resource at each time slot $t \in [T]$ is a function of the amount of virtual BS instances that has been sold:

$$s(y(t)) = \begin{cases} f'(\delta y(t)), & y(t) \le \dfrac{C}{\delta} \\ f'(C)e^{\sigma(y(t)-\frac{C}{\delta})}, & y(t) > \dfrac{C}{\delta} \end{cases} \qquad (6)$$

with parameters

$$\delta = \max\{2, (1+\gamma)^{\frac{1}{\gamma}}\},$$

$$\sigma = \max\{\frac{\gamma(1+2\gamma)}{C(1+\gamma)}, \frac{\delta}{C(\delta-1)} \ln(\frac{U}{\beta_1(1+\gamma)C^\gamma + \beta_2})\},$$

where $U = \max_{n\in[N]} \frac{b_n}{\sum_{m\in[M]} \sum_{t\in[t_n^-, t_n^+]} r_{n,m}(t)}$ is the maximum value per unit of resource per unit of time. In the next subsection, we prove that this price function helps to achieve a good social welfare.

3) Online Auction Mechanism: Following the above discussion, Alg. 1 is the online auction, with the winner determination algorihtm given in Alg. 2. It is based on the classic primal-dual technique, simultaneously solving the primal problem (2) and its dual (3). Lines 1-2 in Alg. 1 define the cost and marginal price function. The values of primal and dual variables are initialized in line 3. Upon the arrival of bid n, Alg. 2 calculates the bidder's utility which is equal to its bidding price minus its requested resources' price, and update the value of dual variable $z_m(t)$. If bidder n has positive utility and appending bid n to the winner set won't violate constraints (2a) and (2c), line 5 in Alg. 2 updates the primal variable x_n by setting it to 1 and increases the amount of hitherto allocated virtual BS instances $y(t)$ for slots $t \in [t_n^-, t_n^+]$. The payment of the winning bid is determined by the marginal price (line 4 in Alg. 2). Line 6 in Alg. 2 updates the marginal price according to the price function (6).

Algorithm 1 A Primal-Dual Online Auction A_{online}

Input: bidding language $\{B_n\}, \{R_m\}, C, \{D_k\}, \beta_1, \beta_2, \gamma$

1: Define cost function $f(y(t))$ according to (1);
2: Define function $s(y(t))$ according to (6);
3: Initialize $x_n = 0, y(t) = 0, z_m(t) = 0, s(t) = 0, w_k(t) = 0, \forall n \in [N], m \in [M], k \in [K], t \in [T]$;
4: **Upon the arrival of the** n**th bid**
5: $\big(x_n, p_n, \{y(t)\}, \{s(t)\}\big) = A_{core}\big(B_n, \{R_m\}, \{D_k\}, \{y(t)\}, \{s(t)\}, s(y(t))\big)$;
6: **if** $x_n = 1$ **then**
7: Accept bid n and allocate resources to bid n according to its demand. Charge bid n p_n;
8: **else**
9: Reject nth bid;
10: **end if**

Algorithm 2 A Winner Determination Algorithm A_{core}

Input: $B_n, \{R_m\}, \{D_k\}, \{y(t)\}, \{s(t)\}, s(y(t))$
Output: $x_n, p_n, \{y(t)\}, \{s(t)\}$

1: $u_n = \max\{0, b_n - \sum_{m\in[M]} \sum_{t\in[t_n^-,t_n^+]} r_{n,m}(t)s(t)\}$;
2: $z_m(t) = u_n / \sum_{m\in[M]} \sum_{t\in[t_n^-,t_n^+]} r_{n,m}(t), \forall m \in [M], \forall t \in [t_n^-, t_n^+]$;
3: **if** $u_n > 0$ and when add n to winning bids' set, constraints (2a) & (2c) are satisfied **then**
4: $p_n = \sum_{m\in[M]} \sum_{t\in[t_n^-,t_n^+]} r_{n,m}(t)s(t)$;
5: $x_n = 1$, $y(t) = y(t) + \sum_{m\in[M]} r_{n,m}(t), t \in [t_n^-, t_n^+;]$;
6: $s(t) = s(y(t)), \forall t \in [t_n^-, t_n^+]$;
7: **end if**
8: **Return** $x_n, p_n, \{y(t)\}, \{s(t)\}$.

B. Theoretical Analysis

1) Correctness, Running time and Truthfulness:

Theorem 1: Upon termination, A_{online} returns a feasible solution for the primal problem in (2) and its dual problem in (3) in $O(NT(M+K))$ time.

Proof: We first examine the correctness of algorithm A_{online}. For the primal problem (2), constraint (2a) and (2c) can be guaranteed due to the `if` condition in Alg. 2, which states that the value of x_n is updated to 1 only when constraints (2a) and (2c) are satisfied. Constraint (2b) holds because line 7 in Alg. 2 guarantees the LHS of (2a) equals $y(t)$ after processing the last bid. Constraint (2d) will not be violated as the value of x_n can only be 0 or 1. For the dual problem (3), since the value of $w_k(t)$ is always zero, constraint (3a) is satisfied because the assignment of $z_m(t)$ ensures that either $\sum_{m\in[M]} \sum_{t\in[t_n^-,t_n^+]} r_{n,m}(t)z_m(t) = b_n - \sum_{m\in[M]} \sum_{t\in[t_n^-,t_n^+]} r_{n,m}(t)s(t)$ when $u_n \geq 0$ or $\sum_{m\in[M]} \sum_{t\in[t_n^-,t_n^+]} r_{n,m}(t)z_m(t) > b_n - \sum_{m\in[M]} \sum_{t\in[t_n^-,t_n^+]} r_{n,m}(t)s(t)$ when $u_n < 0$.

We next analyze the running time of A_{online}. Lines 1-3 can be executed in constant time if we assume that the derivative of cost function $f'(y)$ can be computed in constant time. Upon the arrival of bid n, A_{core} first calculates the value of u_n and $z_m(t)$ in $O(MT)$ time. Because when we check constraints (2a) and (2c), the amount of allocated resources is known, then checking the `if` condition (line 3 in Alg. 2) takes $O(T(M+K))$ steps. The body of the `if` statement (lines 4-6) in Alg. 2 to calculate the payment and update the primal and dual variables can accomplished in $O(MT)$ time. After running A_{core}, the `if-else` statement in Alg. 1 takes constant time to implement the auction result. In conclusion, the overall running time of A_{online} to process N bids is $O(NT(M+K))$. □

Theorem 2: A_{online} is a truthful auction.

Proof: *(Truthfulness in bidding price):* According to Algorithm A_{online}, we can observe that the payment that a bid n needs to pay to the tower company if it wins, depends only on the amount of allocated resources prior to bid n and the amount of requested resources in bid n. If the bidding price of bid n is higher than its payment, the bid is accepted by the tower company. Therefore, our mechanism belongs to the category of *posted pricing mechanisms* [11]. The calculation of the payment for a winning bid is independent of its bidding price. A bidder's utility cannot be improved by misreporting its true valuation as $u_n = v_n - p_n$, and our auction guarantees bidders always achieve their maximum utilities by truthful bidding.

(Truthfulness in arrival time): The bidder cannot arrive earlier as the arrival time is the first time it is aware of its demand. On the other hand, the marginal price increases with the amount of allocated resources. If the bidder delay its arrival time, the amount of the allocated resources also grows, resulting in a higher payment. Therefore, the bidder will never delay its arrival time.

(Truthfulness in required resource amount and resource occupation time): If the operator bids a smaller resource amount and a shorter resource occupation time, it will risk failing to cover the demand hikes. No bidder has incentive to do so. On the other hand, the payment is equal to the total amount of required resources during the occupation time period times the marginal price. So bidding a larger set and a longer time will lead to a higher payment and decrease the bidder's utility. □

2) Competitive Ratio: We next examine the competitive ratio of our online auction, *i.e.,* the upper bound ratio of the optimal social welfare to the social welfare achieved by A_{online}. We first provide the primal-dual analysis framework in Lemma 1. We show that if the gap between the increment of the primal objective function and the dual objective function during the each round is bounded, then we can obtain a bounded competitive ratio.

Let P^n and D^n denote the primal objective value and the dual objective value returned by A_{online} after handling bid n, respectively. Note that P^N and D^N are the final objective values after handling all the bids. $P^0 = 0$ and $D^0 = 0$ due to the initial value of the primal and dual variables, then we have the following lemma:

Lemma 1: If there exists a constant $\alpha \geq 1$ such that $P^n - P^{n-1} \geq \frac{1}{\alpha}(D^n - D^{n-1})$ for the nth round of A_{online} ($\forall n \in [N]$), then the competitive ratio of A_{online} is α.

Proof: By summing up the inequality of each round, it is easy to prove that $P^N = \sum_n (P^n - P^{n-1}) \geq \frac{1}{\alpha}(D^n - D^{n-1}) = \frac{1}{\alpha} D^N$. According to weak duality [33], $D^N \geq OPT$ where OPT is the optimal objective value of convex program (2), we have $P^N \geq \frac{1}{\alpha} OPT$. Therefore A_{online} is α-competitive. □

In order to utilize Lemma 1 to prove the competitive ratio, we next define a Payment-cost Relationship, and verify that the Payment-cost Relationship is a sufficient condition to guarantee the inequality in Lemma 1.

Definition The Payment-cost Relationship for a given parameter $\alpha \geq 1$ is $s^{n-1}(t)\big(y^n(t) - y^{n-1}(t)\big) - \big(f(y^n(t)) - f(y^{n-1}(t))\big) \geq \frac{1}{\alpha}\big(f^*(s^n(t)) - f^*(s^{n-1}(t))\big), \forall n \in [N], \forall t \in [t_n^-, t_n^+]$.

Lemma 2: If the Payment-cost Relationship holds for $\alpha \geq R_{max}$ with $R_{max} = \max_{n,m,t}\{\frac{R_m}{r_{n,m}(t)}\}$, then $P^n - P^{n-1} \geq \frac{1}{\alpha}(D^n - D^{n-1})$ is satisfied for all $n \in [N]$.

Proof: If bid n is rejected, then $P^n - P^{n-1} = D^n - D^{n-1} = 0$. Next, we consider the case when bid n is accepted in the nth round. The increment of the primal objective value after handling bid n is:

$$\begin{aligned} P^n - P^{n-1} &= b_n - \sum_{t \in [t_n^-, t_n^+]} \big(f(y^n(t)) - f(y^{n-1}(t))\big) \\ &= \sum_{m \in [M]} \sum_{t \in [t_n^-, t_n^+]} \big(r_{n,m}(t) z_m^n(t) + r_{n,m}(t) s^{n-1}(t)\big) \\ &- \sum_{t \in [t_n^-, t_n^+]} \big(f(y^n(t)) - f(y^{n-1}(t))\big) \end{aligned}$$

The second equality holds because the value of $w_k(t)$ is always zero and lines 6-7 in A_{online} update the value of dual variables such that dual constraint becomes tight. It is clear that $\sum_{k \in [K]} \sum_{t \in [T]} D_k w_k(t) = 0$, then the increase of the dual objective value is: $D^n - D^{n-1} = \sum_{m \in [M]} \sum_{t \in [t_n^-, t_n^+]} R_m \big(z_m^n(t) - z_m^{n-1}(t)\big) + \sum_{t \in [t_n^-, t_n^+]} \big(f^*(s^n(t)) - f^*(s^{n-1}(t))\big)$. Recall that $y^n(t) - y^{n-1}(t) = \sum_{m \in [M]} r_{n,m}(t)$. Summing up the Payment-cost Relationship over $t \in [t_n^-, t_n^+]$, we obtain:

$$\begin{aligned} P^n - P^{n-1} &\geq \frac{1}{\alpha}(D^n - D^{n-1}) + \sum_{m \in [M]} \sum_{t \in [t_n^-, t_n^+]} r_{n,m}(t) z_m^n(t) \\ &- \frac{1}{\alpha} \sum_{m \in [M]} \sum_{t \in [t_n^-, t_n^+]} R_m \big(z_m^n(t) - z_m^{n-1}(t)\big) \end{aligned}$$

Since $\alpha \geq \max_{n,m,t}\{\frac{R_m}{r_{n,m}(t)}\}$ and $z_m^n(t) \geq 0$,

$$\sum_{m \in [M]} \sum_{t \in [t_n^-, t_n^+]} r_{n,m}(t) z_m^n(t) \geq \frac{1}{\alpha} \sum_{m \in [M]} \sum_{t \in [t_n^-, t_n^+]} R_m z_m^n(t)$$

$$\begin{aligned} \Rightarrow &\sum_{m \in [M]} \sum_{t \in [t_n^-, t_n^+]} r_{n,m}(t) z_m^n(t) - \frac{1}{\alpha} \sum_{m \in [M]} \sum_{t \in [t_n^-, t_n^+]} R_m z_m^n(t) \\ &+ \frac{1}{\alpha} \sum_{m \in [M]} \sum_{t \in [t_n^-, t_n^+]} R_m z_m^{n-1}(t) \geq 0 \end{aligned}$$

As a result, $P^n - P^{n-1} \geq \frac{1}{\alpha}(D^n - D^{n-1})$ and we finish the proof. □

The differential version of the Payment-cost Relationship is defined as below. Lemma 3 presents the value of α and β_2 such that they satisfy the Differential Payment-cost Relationship, based on Lemma 4 and Lemma 5.

Definition The Differential Payment-cost Relationship for a given parameter $\alpha \geq 1$ is

$$s(t) dy(t) - f'(y(t)) dy(t) \geq \frac{1}{\alpha} f^{*\prime}(s(t)) ds(t), \quad \forall n \in [N], \forall t \in [t_n^-, t_n^+]. \tag{7}$$

Lemma 3: The marginal payment function defined in (6) satisfies the Differential Payment-cost Relationship defined in (7) for

$$\alpha = \max\{R_{max}, 4(1+\gamma), \frac{2(1+2\gamma)}{\gamma} \ln(\frac{U}{\beta_1(1+\gamma)C^\gamma + \beta_2})\}$$

and $\beta_2 \leq \gamma C^\gamma \beta_1$.

Proof: We first provide the explicit expressions for the differentials of (1) and its convex conjugate (5):

$$f'(y(t)) = \begin{cases} \beta_1(1+\gamma) y(t)^\gamma + \beta_2, & \text{if } y(t) \in [0, C] \\ +\infty, & \text{otherwise} \end{cases}.$$

$$f^{*\prime}(s(t)) = \begin{cases} \Big(\dfrac{s(t) - \beta_2}{\beta_1(1+\gamma)}\Big)^{\frac{1}{\gamma}}, & s(t) \leq \beta_!(1+\gamma)C^\gamma + \beta_2 \\ C, & s(t) > \beta_1(1+\gamma)C^\gamma + \beta_2 \end{cases}.$$

When the amount of allocated virtual BS instances reaches the capacity of virtual BS pool, *i.e.,* $y(t) = C$, according to the value of σ and the definition of marginal payment in (6), $s(t) = (\beta_1(1+\gamma)C^\gamma + \beta_2) e^{\sigma(C - \frac{C}{\delta})} \geq U$. Recall that U is the maximum value per unit of resource per unit of time. It is clear when the marginal price is larger than U, no bids can win. Thus, we may assume $y(t) \leq C$ in the rest of the proof, and $f'(y(t)) = \beta_1(1+\gamma) y(t)^\gamma + \beta_2$. Next, we divide our proof into two cases:

Case 1: $y(t) \leq \frac{C}{\delta}$: Because $s(t) = f'(\delta y(t)) = \beta_1(1+\gamma)(\delta y(t))^\gamma + \beta_2 \leq \beta_!(1+\gamma)C^\gamma + \beta_2$, the Differential Payment-cost Relationship can be rewritten as:

$$\begin{aligned} &(\beta_1(1+\gamma)\delta^\gamma y(t)^\gamma - \beta_1(1+\gamma) y(t)^\gamma) dy(t) \\ &\geq \frac{1}{\alpha} \Big(\frac{\beta_1(1+\gamma)\delta^\gamma y(t)^\gamma}{\beta_1(1+\gamma)}\Big)^{\frac{1}{\gamma}} \beta_1(1+\gamma)\delta^\gamma \gamma y(t)^{\gamma-1} dy(t). \end{aligned} \tag{8}$$

Cancelling the common term on both sides, (8) becomes $(\delta^\gamma - 1) \geq \frac{1}{\alpha} \gamma \delta^{\gamma+1}$. **i)** If $\gamma \geq 1$, $\delta = \max\{2, (1+\gamma)^{\frac{1}{\gamma}}\} = 2$, we can obtain $\frac{\gamma \delta^{\gamma+1}}{\delta^\gamma - 1} = \frac{\gamma 2 \cdot 2^\gamma}{2^\gamma - 1} = \frac{\gamma(4 \cdot 2^\gamma - 2 \cdot 2^\gamma)}{2^\gamma - 1} \leq \frac{4\gamma(2^\gamma - 1)}{2^\gamma - 1} \leq 4\gamma < \alpha$ **ii)** If $\gamma < 1$, then $\delta = (1+\gamma)^{\frac{1}{\gamma}} < e$, and $\frac{\gamma \delta^{\gamma+1}}{\delta^\gamma - 1} = \delta(1+\gamma) < e(1+\gamma) < \alpha$.

Case 2: $y(t) > \frac{C}{\delta}$: In this case, the marginal payment $s(t) = (\beta_1(1+\gamma)C^\gamma + \beta_2) e^{\sigma(y(t) - \frac{C}{\delta})}$. Note that $ds(t) = \sigma s(t) dy(t)$, then the Differential Payment-cost Relationship is:

$$(s(t) - f'(y(t)))dy(t) \geq \frac{1}{\alpha} C\sigma s(t)dy(t). \quad (9)$$

By lemma 4, we can obtain $s(t) - f'(y(t)) \geq s(t) - \frac{1+\gamma}{1+2\gamma}s(t) = \frac{\gamma}{1+2\gamma}s(t)$, thus to prove (9), it is sufficient to prove: $\frac{\gamma}{1+2\gamma}s(t)dy(t) \geq \frac{1}{\alpha}C\sigma s(t)dy(t) \Rightarrow \sigma \leq \frac{\gamma}{C(1+2\gamma)}\alpha$. By the setting of σ, either **i)**

$$\sigma = \frac{\gamma(1+2\gamma)}{C(1+\gamma)} = \frac{\gamma}{C(1+2\gamma)}\frac{(1+2\gamma)^2}{1+\gamma} < \frac{\gamma}{C(1+2\gamma)}\frac{(2+2\gamma)^2}{1+\gamma}$$
$$= \frac{\gamma}{C(1+2\gamma)}4(1+\gamma) \leq \frac{\gamma}{C(1+2\gamma)}\alpha.$$

or **ii)** $\sigma = \frac{\delta}{C(\delta-1)}\ln(\frac{U}{\beta_1(1+\gamma)C^\gamma+\beta_2})$

$$\leq \frac{2}{C}\ln(\frac{U}{\beta_1(1+\gamma)C^\gamma+\beta_2})$$
$$= \frac{\gamma}{C(1+2\gamma)}\frac{2(1+2\gamma)}{\gamma}\ln(\frac{U}{\beta_1(1+\gamma)C^\gamma+\beta_2}) \leq \frac{\gamma}{C(1+2\gamma)}\alpha.$$

In conclusion, we have finished the proof for both cases. □

Lemma 4: When $y(t) > \frac{C}{\delta}$, the marginal payment $s(t)$ is larger than the marginal cost by a factor of at least $\frac{1+2\gamma}{1+\gamma}$: $s(t) \geq \frac{1+2\gamma}{1+\gamma}f'(y(t))$.

Proof: When $y(t) > \frac{C}{\delta}$, $s(t) = (\beta_1(1+\gamma)C^\gamma + \beta_2)e^{\sigma(y(t)-\frac{C}{\delta})}$. So Lemma 4 is equivalent to verify

$$(\beta_1(1+\gamma)^2C^\gamma + (1+\gamma)\beta_2)e^{\sigma y(t)}$$
$$- \beta_1(1+\gamma)(1+2\gamma)y(t)^\gamma e^{\frac{\sigma C}{\delta}} \geq \beta_2(1+2\gamma)e^{\frac{\sigma C}{\delta}}. \quad (10)$$

We first show that the inequality (10) holds when $y(t) = \frac{C}{\delta}$. If $y(t)$ takes the value of $\frac{C}{\delta}$, (10) becomes $\beta_2 \leq \frac{1+\gamma}{\gamma}(1+\gamma - \frac{1+2\gamma}{\delta^\gamma})C^\gamma\beta_1$, which is obviously true according to Lemma 5.

Next, it suffices to show the left side of (10) is non-decreeing as $y(t)$ increases. Let $E(y(t))$ denote the left hand of (10). The derivative of $E(y(t))$ is: $E'(y(t)) = \sigma(\beta_1(1+\gamma)^2C^\gamma + (1+\gamma)\beta_2)e^{\sigma y(t)} - \beta_1(1+\gamma)(1+2\gamma)\gamma y(t)^{\gamma-1}e^{\frac{\sigma C}{\delta}}$. Since $\sigma \geq \frac{\gamma(1+2\gamma)}{C(1+\gamma)}$, $\frac{C}{\delta} < y(t) \leq C$ and $(1+\gamma)\beta_2 \geq 0$, then $\sigma(\beta_1(1+\gamma)^2C^\gamma + (1+\gamma)\beta_2)e^{\sigma y(t)}$ is larger than $\beta_1(1+\gamma)(1+2\gamma)\gamma y(t)^{\gamma-1}e^{\frac{\sigma C}{\delta}}$ and the derivative $E'(y(t))$ is nonnegative. Consequently, the lemma follows. □

Lemma 5: The value of β_2 defined in Lemma 3 ($\beta_2 \leq \gamma C^\gamma\beta_1$) satisfies the inequality $\beta_2 \leq \frac{1+\gamma}{\gamma}(1+\gamma-\frac{1+2\gamma}{\delta^\gamma})C^\gamma\beta_1$.

Proof: The value of γ divides the proof into two cases:

Case 1: If $0 < \gamma < 1$, $\delta^\gamma = (1+\gamma)^{\frac{1}{\gamma}\gamma} = 1+\gamma$. Evidently, $\beta_2 \leq \gamma C^\gamma\beta_1 = \frac{1+\gamma}{\gamma}\frac{\gamma^2}{1+\gamma}C^\gamma\beta_1$, which indicates the inequality holds.

Case 2: Otherwise, $\delta^\gamma = 2^\gamma (\gamma \geq 1)$, WolframAlpha [34] states that

$$\min\{\frac{1+\gamma}{\gamma}(1+\gamma-\frac{1+2\gamma}{2^\gamma}) - \gamma | \gamma \geq 1\} = 0 \ at \ \gamma = 1.$$

Therefore, $\frac{1+\gamma}{\gamma}(1+\gamma-\frac{1+2\gamma}{2^\gamma}) \geq \gamma$, which means if $\beta_2 \leq \gamma C^\gamma\beta_1$ holds, $\beta_2 \leq \frac{1+\gamma}{\gamma}(1+\gamma-\frac{1+2\gamma}{\delta^\gamma})C^\gamma\beta_1$ must also hold. In summary, we have finished the proof. □

Theorem 3: The online auction mechanism A_{online} achieves at least $\frac{1}{\alpha}$ of the optimal social welfare, with α and β_2 given in Lemma 3.

Proof: By lemma 3, the marginal payment function defined in (6) satisfies the Differential Payment-cost Relationship in (7) with parameters α and β_2. Next, we assume that $y^n(t) - y^{n-1}(t) = \sum_{m\in[M]} r_{n,m} = dy(t)$ is very small compared to the capacity of the virtual BS pool C, then $f(y^n(t)) - f(y^{n-1}(t)) = f'(y(t))dy(t), f^*(s^n(t)) - f^*(s^{n-1}(t)) = f^{*\prime}(s(t))ds(t)$. The above equations imply that the Payment-cost Relationship holds for parameters α and β_2. Combining Lemma 1 and Lemma 2, we verify that the online auction mechanism A_{online} is α-competitive in social welfare. □

C. Discussion

In Sec. III and Sec. IV-A, we have focused on the model where the tower company owns the entire C-RAN infrastructure. In a future 5G network, it is also conceivable that the tower company opts to rent mobile data services from dedicated cloud service providers, instead of constructing its own data centers. It can rent virtual machines for signal processing from cloud service providers such as Amazon or Google. In this case, the cost function (1) becomes somewhat different and can be rewritten as:

$$f(y(t)) = \begin{cases} \beta_1 y(t) + \beta_2 y(t), & \text{if } y(t) \in [0, C] \\ +\infty, & \text{otherwise} \end{cases} \quad (11)$$

Similar to the pervious cost function, $\beta_1 y(t)$ is the cost of the virtual BS pool, which is linear to the number of virtual BS instances purchased from the cloud service provider. We can observe that (11) is a special case of (1) with $\gamma = 0$, the auction design and the analysis are similar to the counterparts in Sec. IV-A and Sec. IV-B, and we omit the details due to space limitation.

V. Performance Evaluation

To examine the performance of our online auction A_{online}, we conduct large-scale simulation studies based on real world data. We first introduce the setting of the C-RAN system. Two representative pilot C-RAN systems constructed in the past three years have 11 BSs and 18 BSs, respectively [3]. Hence, the number of BSs (M) in our simulations is an integer chosen from $[10, 30]$. With reference to the number of resource blocks in the LTE channel [35], we set the amount of available spectrum resources at each BS R_m to an integer within $[40, 100]$. The capacity of the virtual BS pool C is a function of the number of base stations (M), and is set within $[15M, 40M]$. The front-haul can have either a point-to-point or a daisy chain topology [26]; in either case, the number of fiber links (K) is less than or equal to the number of BSs (M). Therefore, we set K to a random number that is less than M. The bandwidth requirement to transmit baseband signal is as high as dozens of Gbps [26], the capacity of each fiber link (D_k) is set within $[10, 20]$ Gbps. For the cost of the virtual BS pool, γ is set within $[0.5, 2.2]$ and $\beta_1 = 0.4$ [29], [30]. Regarding spectrum price and RRHs' transmission cost, we let $\beta_2 = 0.5$ as our algorithm requires $\beta_2 \leq \gamma C^\gamma\beta_1$ and the cost is moderate to occupy a unit spectrum to transmit data in a short period. The C-RAN auction spans 24 hours ($T = 1440$ mins) with each slot being 1 min. A bid arrives every 1 to 5 slots, requesting to occupy $[5, 60]$ slots. By default, $r_{n,m}(t) \in [2, 10]$ and $d_{n,k}(t) \in [0, 4]$. The total number of bids N is set within

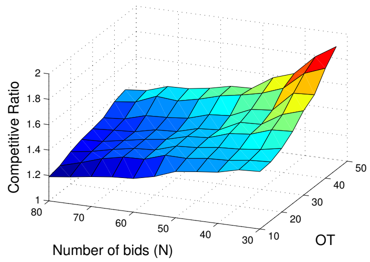

Fig. 2. Competitive ratio with different number of bids and occupying time

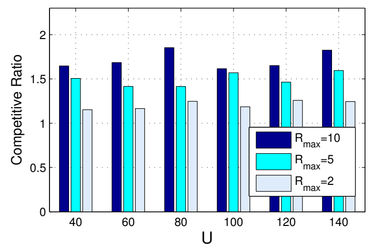

Fig. 3. Competitive ratio with U and R_{max}.

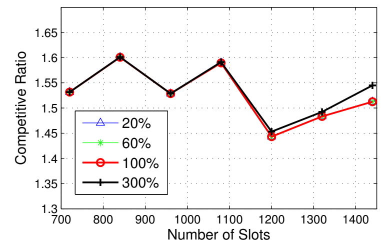

Fig. 4. Competitive ratio with estimated U and different T.

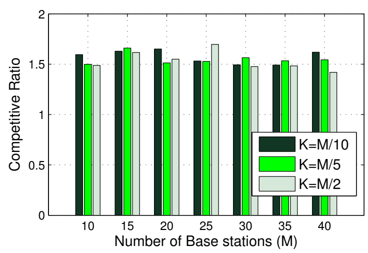

Fig. 5. Competitive ratio with different number of BSs and fiber links.

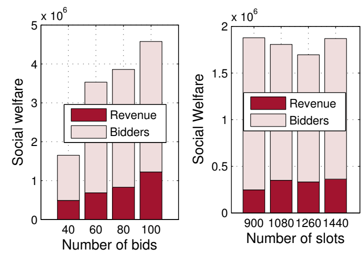

Fig. 6. Bidders' uiltity vs. tower company's revenue under different N and T.

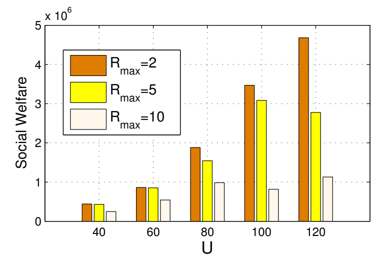

Fig. 7. Social welfare with U and R_{max}.

$[30, 100]$. The bidding price is equal to the overall resource demand in the bid times a unit price randomly picked in a range, between the lower and upper bounds of bidders' value per unit resource per slot. The default values of the two bounds are 10 and 80 respectively. In order to improve the reliability of the simulation result, we repeat each set of simulations 20 times, and use the average result to plot the corresponding figure.

Competitive Ratio. We first study the competitive ratio achieved by our online algorithm A_{online}. We use `CVX` with the Gurobi Optimizer to solve the convex program (2) exactly, and compute the competitive ratio by dividing the optimal social welfare by the social welfare returned by A_{online}.

Fig. 2 shows the competitive ratio under different number of bids with different average occupying time $(t_n^+ - t_n^-)$. The competitive ratio decreases as the number of bids grows, and increases when the average occupying time rises. As the number of bids grows, the algorithm has a larger solution space to work with, and it is less likely that it will commit to low-value bids prematurely. Furthermore, short occupying time means less competition in each time slot and lower inter-slot correlation in decision making, making it easier for the algorithm to make optimized decisions. Fig. 3 reveals that a better performance of A_{online} comes with a small value of R_{max}, while the value of U has only minor influence on the competitive ratio. This can be explained as following. As indicated in Lemma 3, the upper bound of the competitive ratio is the largest number between three parameters. Because $\beta_1(1+\gamma)C^\gamma + \beta_2$ is much larger than U in the third parameter, a change in U does not influence the value of the ratio. R_{max} is one of the parameters that determine the ratio. In Fig. 4, we use the estimate of U as the input to A_{online}, to investigate the performance of our algorithm at different estimation levels with different number of slots. Neither under-estimation or over-estimation matters much to the performance. Over-estimation is slightly worse than under-estimation, as compared to the real U (labelled by 100%). This is because when we use an over-estimated U as an input to calculate the unit resource price $s(t)$, $s(t)$ becomes larger than its actual value and hence the resource price is increased improperly. As a result, bids that should be accepted are rejected, leading to a worse performance. The online auction works well for any number of time slots and there in no upward or downward trend with the increase of T, which is inline with competitive ratio analysis in Theorem 3 which indicates that the competitive ratio α doesn't depend on the value of T. Fig. 5 demonstrates the competitive ratio with different number of BSs and fiber links. Our algorithm always performs well with a small competitive ratio (< 2) under any M and K. In addition, the size of the C-RAN in terms of the number of BSs and fiber links doesn't affect the performance of our online auction. Again, the observation in Fig 5 confirms the analysis in Theorem 3, which shows that only three parameters dominate the ratio.

Social Welfare. We next evaluate the performance of A_{online} in terms of social welfare. Fig. 6 shows the social welfare achieved by A_{online} under different setting. The left figure illustrates that a larger number of bids leads to a higher social welfare and higher seller revenue. This is because a larger set of bids provides more high value bids that tower company can select. The right figure demonstrates the fluctuation of the social welfare with the number of slots changes. We can observe that the value of T doesn't influence the social welfare or the tower company's revenue, both of which are determine by the bidding price and the operation cost. However, the value of U and R_{max} have impact on social welfare. Fig. 7 shows that social welfare drops sharply when the maximum value per unit of resources per slot (U) decreases. We know the bidding price is set according to the value of U; thus, the bidding prices drop accordingly with the decrease of U, leading to a lower social welfare. Furthermore, under a small

R_{max}, A_{online} achieves a higher social welfare. This is in line with Lemma 3, which proves that the performance of A_{online} is limited by the value of R_{max}.

Percentage of Winners. Finally, we investigate winner satisfaction as measured by the percentage of winners. Fig. 8 presents the fact that a higher fraction of bids are accepted when a smaller number of bids are submitted during the C-RAN auction. This is because the number of winners is limited by the resource capacity and is almost fixed. Furthermore, the length of the resource occupying time influences the percentage of winners. Fig. 9 reflects an upward trend as the capacity of virtual BS pool (C) grows. However, when C is large enough ($C \geq 30M$), the percentage of winners fluctuates around a fixed level.

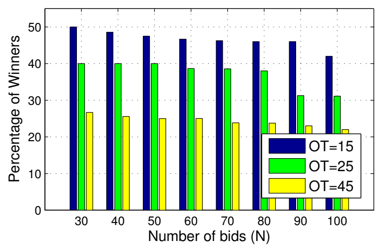

Fig. 8. Percentage of winners under different number of bids and occupying time.

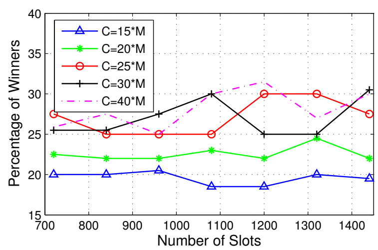

Fig. 9. Percentage of winners under different T and C.

VI. Conclusions

As the transition from LTE and LTE-A to 5G wireless networks progresses and engineering advancements mature, unique economic challenges in the 5G paradigm are starting to attract attention. This work studies the 5G market under the C-RAN architecture, where mobile operators arrive online and lease resources from the infrastructure owner, the tower company. Combining recent techniques of Fenchel dual for convex optimization and posted pricing online auction design, we present a C-RAN auction that executes efficiently in an online fashion, guarantees truthfulness, and approximately maximizes social welfare. As a future direction, it will be interesting to study the C-RAN market with more fine-grained correspondence between spectrum at BSs and BS instances at the mobile cloud.

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