Anchor Selection for Localization in Large Indoor Venues

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***Abstract*—Many indoor localization systems rely on a set of reference anchors with known positions. A target's location is estimated from a set of distances between the target and its surrounding anchors, and hence the selection of anchors affects the localization accuracy. However, it remains a challenge to select the best set of anchors. In this paper, we study how to appropriately make use of the surrounding anchors for localizing a target. We first construct different candidate anchor clusters by selecting different number of anchors with the strongest received signals. Then for each candidate cluster, we propose a weighted min-max algorithm to provide a location estimation. Finally, we introduce a weighted geometric dilution of precision (w-GDOP) algorithm that combines the estimations from multiple clusters by quantifying their estimation accuracy. We evaluate the performance of our solution through simulations and real-world experiments. Our results show that the proposed anchor selection scheme and localization algorithm significantly improve the localization accuracy in large indoor environments.**

I. Introduction

In recent years, wireless localization has received significant attention in both academia and industry, with the advent of emergency services and location-based services in wireless sensor networks (WSNs) [1]–[3]. In WSN, sensor nodes have the ability of sensing environmental parameters and are equipped with limited resources for communication and computation. Due to these capabilities, they can be applied to perform specific tasks independently under different environmental conditions. However, sensing data without knowing the node's location is quite inadequate; thus investigating approaches for estimating the location of a sensor from input communications with other sensors becomes a crucial issue. In localization, the locations of a set of anchors are known in advance. To determine the location of a target, it first estimates the distances to at least three surrounding anchors, based on which its own location can then be estimated by a localization algorithm. Given a large number of surrounding anchors, it is crucial to determine which anchors a target should engage in localization to reduce redundancy in communication and computation while estimating the target's location accurately.

In localization, accurate anchor selection techniques to effectively create clusters and efficiently evaluate each cluster to determine the most accurate cluster directly impact the quality of services provided by the WSN. Therein, indices and limits such as mean squared error (MSE), cumulative probability distribution and mean variance of the error have been proposed to evaluate different anchor selection techniques. Most recently, techniques which take into consideration the geometric factors of the anchors such as the Cramér-Rao lower bound (CRLB) and geometric dilution of precision (GDOP) have been introduced to enhance selection. This is due to their ability to separate geometric factor from the localization error, and as a result they show superior performance in WSNs.

In this paper, we address the anchor selection problem so as to achieve a more accurate target localization to enhance the positioning in large indoor venues. First, we propose a simple technique for building candidate anchor clusters based on the received signal strength (RSS). The rationale is that low RSSs result in large errors in the estimated distances and hence reduce the localization accuracy. We then propose a weighted min-max localization algorithm to overcome the deficiency in the traditional min-max algorithm. For each candidate anchor cluster, we use the weighted min-max algorithm to calculate an estimated location. Finally, we design a weighted GDOP technique to quantify the estimation accuracy of different candidate clusters, and obtain the final estimate of targets by combining the estimates from multiple anchor clusters. The performance of our proposed solution has been evaluated by computer simulations and also real-world experiments. The simulation and experimental results show that our proposed solution outperforms several recent anchor selection solutions.

The rest of the paper is organized as follows. In Section II, we summarize the related works and in Section III, we briefly present the preliminaries for target localization. In Section IV, we present our anchor selection strategy and a weighted min-max algorithm for estimating the target location based on a single candidate anchor cluster. In Section V, we introduce the weighted GDOP algorithm for evaluating different anchor clusters and then discuss how we obtain the final location estimation from multiple anchor clusters. Analysis through simulations and real-world experiments is provided in Section VI. Finally, conclusions are drawn in Section VII.

II. Related Work

In the literature, the problem of anchor selection has received wide attention recently [7]–[10]. In [7], an efficient GDOP calculation method was proposed by applying matrix multiplication to enhance location estimation in positioning systems. In [8], a node-selection strategy was proposed. Here, A-CRLB value for each node is calculated to represent the contribution values that the node makes to the localization accuracy. In [9], a cooperative localization algorithm with a CRLB-based cluster nodes selection strategy was proposed to determine the nodes that make the greatest contribution to the localization accuracy. In [10], Ahmadi et al proposed k-nearest neighbor classifier to select anchors nearest to the target and applied a regression tree-based localization algorithm on the selected anchors.

Generally, these works aim at complex numerical calculations rather than a relatively simple expression. As such, the large amount of calculations may lead to increased communication traffic and energy consumption.

III. Preliminaries

A. System Model

Consider a wireless network of $N + M$ nodes in an η-dimensional space, out of which nodes indexed $1, \cdots, N$ are the *targets* with unknown locations, while nodes indexed $N + 1, \cdots, N + M$ are *anchors* with *a priori* known locations.

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First, we estimate the set of distances between a target and anchors using the received signal strengths (RSSs), which is called measured distance. Then the location of the target node can be estimated by the knowledge of the anchor locations and the set of measured distances. To elaborate, the coordinates of the well-known anchors are described by

$$\text{Input}: \ \boldsymbol{\Phi} \triangleq \big[\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \cdots, \boldsymbol{\varphi}_M\big]. \tag{1}$$

Likewise, the coordinates of the targets to be estimated are

$$\text{Output}: \ \boldsymbol{\Theta} \triangleq \big[\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \cdots, \boldsymbol{\theta}_N\big]. \tag{2}$$

The set of true distances between the targets and anchors are

$$\mathcal{D} \triangleq \begin{bmatrix} \mathbf{d}_1 \\ \mathbf{d}_2 \\ \vdots \\ \mathbf{d}_N \end{bmatrix} = \begin{bmatrix} d_{11} & \cdots & d_{1M} \\ d_{21} & \cdots & d_{2M} \\ \vdots & \cdots & \vdots \\ d_{N1} & \cdots & d_{NM} \end{bmatrix}. \tag{3}$$

The measured distances are typically affected by signal noise and a positive *bias*. Therefore, the ranging model applicable to the n-th target and m-th anchor is

$$\tilde{d}_{nm} = d_{nm} + e_{nm} = \big\|\boldsymbol{\varphi}_m - \boldsymbol{\theta}_n\big\| + e_{nm}, \tag{4}$$

where $\tilde{d}_{nm}$ is the measured distance, d_{nm} is the true distance and e_{nm} is an additive *residual noise* in the wireless channel.

B. RSSI-based Ranging

We use the log-normal shadow model for the RSS which is suitable for both indoor and outdoor environments [4]. In this model, the received power at a distance d from the transmitter is

$$P_r(d) = P_t - 10\log_{10}\left(\frac{4\pi d_0}{\lambda}\right)^2 - 10\alpha\log_{10}\frac{d}{d_0} + X_\sigma, \tag{5}$$

where P_t is the transmitted power, d_0 is the reference distance (1m), α is the path loss exponent and X_σ is a zero-mean gaussian random variable with standard deviation σ, all in dB. Due to the relationship between the distance and RSS in Eq. (5), the measured distance $\tilde{d}_{nm}$ is obtained by

$$\tilde{d}_{nm} = 10^{\dfrac{P_{t:m} - P_r(d_{nm})}{10\alpha}}. \tag{6}$$

IV. Anchor Placement, Anchor Selection And Weighted Min-Max Algorithm

A. Anchor Placement

The location of anchors plays a key role in the estimation error of targets to be localized. By studying the MSE of an unbiased estimator, it is possible to recognize the impact of anchors placement in the localization problem. Therefore, efficient placement of anchors for easy localization leading to the minimization of the MSE of all targets is very crucial. In [12], tight frames due to their resilience and robustness to additive white noise lead to the minimization of the mean squared error in targets estimation.

Tight frames as a signal representation tool can represent any vector in the Euclidean space. A finite sequence of linearly dependent vectors $\boldsymbol{\Phi} = \{\boldsymbol{\varphi}_m\}_{m=1}^{M}$ in an Euclidean space $\mathbb{R}^\eta$, for $M > \eta$, is called a **tight frame** if there exist constants $A = B$ such that for any $\boldsymbol{\theta}_n$ in $\mathbb{R}^\eta$,

$$A\big\|\boldsymbol{\theta}_n\big\|^2 = \sum_{m=1}^{M}\Big|\big\langle\boldsymbol{\varphi}_m, \boldsymbol{\theta}_n\big\rangle\Big|^2 = B\big\|\boldsymbol{\theta}_n\big\|^2 \ \ \forall \ \ n, \tag{7}$$

where A, B are the **frame bounds** related to stability [13]. In this paper, tight frames as a generalization of the orthonormal basis will be used in efficiently placing anchors for target localization. By spreading the input observations over a wider range of anchor vectors $\boldsymbol{\Phi}$, we can achieve an increase in resilience to mitigate the effects of noise in target estimates.

B. Anchor Selection

For a target, we can select a cluster of anchors to provide an estimation of the target location. How to select the clusters of anchors for location estimation is known as the anchor selection problem [7]–[10]. Here, we propose to build up multiple clusters for a single target to provide multiple estimations, based on which a more accurate estimation can be obtained. In this subsection, we first establish some criteria with references for selecting clusters of anchors that a target should cooperate with so as to achieve a more accurate localization. We propose a simple technique for selecting clusters of anchors based on the target-to-anchor signals as follows:

- Cluster based on strong RSS measurements – anchor clusters for target localization are created by selecting anchors with the strongest RSSs between the anchors and the target. In a low-power wirelessly networked system, under the same channel conditions anchors with stronger RSS lead to better network connections [2]. This anchor selection solution can be applied to off-the-shelf hardware such as WiFi access points without changing the firmware.

First, our idea is to select anchors with strong RSSs. However, the RSS values of each pair of target n and anchor m are usually dynamic. Therefore, we use the median of RSS $\tilde{r}_{nm}$ within a time window μ to sort the anchors, which is more stable than the mean in avoiding outliers.

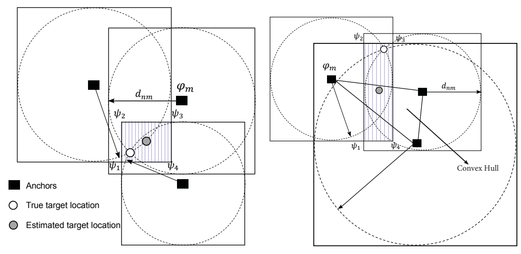
Here, a unique representation of the anchor clusters is

$$\begin{aligned} C_i(\boldsymbol{\Phi}^-) = &\Big\{\boldsymbol{\varphi}_{\tilde{r}_{n1}} > \boldsymbol{\varphi}_{\tilde{r}_{n2}} > \cdots > \boldsymbol{\varphi}_{\tilde{r}_{nM_i^-}}\Big\}, \\ &\forall \ \ i = 1, \cdots, c, \end{aligned} \tag{8}$$

where each cluster $C_i(\boldsymbol{\Phi}^-)$ contains a subset of anchors $\boldsymbol{\Phi}^- \subset \boldsymbol{\Phi}$ based on the M_i^- strongest RSS $\boldsymbol{\varphi}_{\tilde{r}_{nm}}$, $M_i^- \geq 3$ is the number of anchors in the i-th cluster, and c is the total number of anchor clusters. To determine c, the RSS $\boldsymbol{\varphi}_{\tilde{r}_{nm}}$ of all anchors in each cluster $C_i(\boldsymbol{\Phi}^-)$ must always be greater than a predefined cut-off RSS threshold $\tilde{r}_{th}$.

C. Weighted Min-Max Algorithm

After anchor cluster selection, we require a localization algorithm to estimate an initial target location $\hat{\boldsymbol{\theta}}_{n:i}$ for each anchor cluster $C_i(\boldsymbol{\Phi}^-) \ \forall \ i = 1, \cdots, c$. For localizing a large number of targets (such as customers in a shopping mall or exhibition hall), the computational time of the localization algorithm is important. Hence, we propose an efficient yet accurate localization algorithm. In the literature, the most common localization algorithms are the least squares algorithms. Unfortunately, these algorithms compute exact solutions of the target's location which assume the existence of error-free distances [14]. In large indoor venues with high NLOS and multipath conditions, these algorithms are not applicable, and therefore, there is a need of a localization algorithm for computing approximate solutions in the presence of large measured distance errors. Therein, we propose a weighted



(a) (b)

Figure 1: Operation in determining the min-max algorithm.

min-max algorithm that can improve the accuracy of the traditional min-max algorithm.

In the traditional algorithm [3], a square is bounded around the target in Figure 1a. The length of the square is twice the measured distance $\tilde{d}_{nm}$ between the target and each anchor. The squares are used to create an area in which the target's location is to be estimated. This estimation area is made up of vertices $\psi_j, j = 1, 2, 3, 4$ placed at the following locations:

$$\begin{aligned}
\psi_1 &= [max(\varphi_{m:x} - \tilde{d}_{nm}), max(\varphi_{m:y} - \tilde{d}_{nm})] \\
\psi_2 &= [max(\varphi_{m:x} - \tilde{d}_{nm}), min(\varphi_{m:y} + \tilde{d}_{nm})] \\
\psi_3 &= [min(\varphi_{m:x} + \tilde{d}_{nm}), min(\varphi_{m:y} + \tilde{d}_{nm})] \\
\psi_4 &= [min(\varphi_{m:x} + \tilde{d}_{nm}), max(\varphi_{m:y} - \tilde{d}_{nm})] \ \forall \ n, m,
\end{aligned} \tag{9}$$

where *min*() and *max*() are the minimum and maximum functions respectively. This area is created by the intersection of the squares resulting in the smallest shaded region around the possible target location. Nonetheless, overlapping might sometime not occur due to lack of connecting squares. This occurs when the measured distances are shorter than the true distances $\tilde{d}_{nm} \ll d_{nm}$. Using the received signal strength as a ranging technique, the measured distance is always larger than the true distance $\tilde{d}_{nm} > d_{nm}$ as seen in Eq. (6), which avoids this scenario.

For the min-max algorithm, the position of the target $\hat{\boldsymbol{\theta}}_{n:i}$ for each cluster $C_i(\boldsymbol{\Phi}^-)$ is estimated at the center of the estimation zone:

$$\hat{\boldsymbol{\theta}}_{n:i} = \frac{\psi_1 + \psi_3}{2} = \frac{\psi_2 + \psi_4}{2}. \tag{10}$$

which is always inside the convex hull of the anchors $\boldsymbol{\Phi}$. Unfortunately, the min-max algorithm can lead to large errors when the target's true location is outside the convex hull of the anchors as shown in Figure 1b. Therefore, the further the target from the convex hull, the larger the errors in $\hat{\boldsymbol{\theta}}_{n:i}$. As seen, the target's location is not necessarily at the center of the estimation area and could be localized anywhere within the area as such the convex hull problem can be minimized. As a result, a weighted min-max algorithm is presented to give weights $w_a(j)$ to the vertices $j = 1, 2, 3, 4$ of the estimation area to solve this problem. In this paper, we present the weighted min-max algorithm using two different weights:

$$\begin{aligned}
w_1(j) &= \frac{1}{\sum_{m=1}^{M} |D_{mj} - \tilde{d}_{nm}|} \\
w_2(j) &= \frac{1}{\left(\sum_{m=1}^{M} |D_{mj} - \tilde{d}_{nm}|^2\right)^{1/2}},
\end{aligned}$$

where $D_{mj} = \left\|\boldsymbol{\varphi}_m - \boldsymbol{\psi}_j\right\|$ is the Euclidean distance between anchor $\boldsymbol{\varphi}_m$ and vertex $\boldsymbol{\psi}_j$ for each anchor cluster. Therefore, the position of the target $\hat{\boldsymbol{\theta}}_n$ is calculated using the obtained weights and the coordinates of the vertices, as such the target's location is estimated as:

$$\hat{\boldsymbol{\theta}}_{n:i} = \left[\frac{\sum_{j=1}^{4} w_a(j) \cdot \psi_{j:x}}{\sum_{j=1}^{4} w_a(j)}, \frac{\sum_{j=1}^{4} w_a(j) \cdot \psi_{j:y}}{\sum_{j=1}^{4} w_a(j)}\right]. \tag{11}$$

V. Weighted Geometric Dilution of Precision For Target Localization

In the previous section, our weighted min-max algorithm can provide an estimate for a given cluster of anchors. However, since there is a set of candidate clusters $C_i(\boldsymbol{\Phi}^-)$ for each target $\boldsymbol{\theta}_{n:i}$, there remains a challenge in finding a better estimate based on the initial estimates $\hat{\boldsymbol{\theta}}_{n:i}$ of all c clusters. To this end, we first propose a weighted geometric dilution of precision (w-GDOP) to estimate the accuracy of each cluster. Also, a new estimate will be calculated based on the initial estimates and their corresponding accuracies.

A. Weighted GDOP

GDOP is a popular method used in satellite systems to state how errors in the distance measurements affect the estimation of target location [9], [11]. The GDOP is defined as the ratio of the position's mean squared error and the distance measurement's mean squared error. A smaller GDOP indicates a smaller mean squared error in the target's estimate (higher accuracy) [7], [11]. Therefore, the GDOP of a single target $\boldsymbol{\theta}_{n:i}$ for each anchor cluster $C_i(\boldsymbol{\Phi}^-)$ can be expressed as

$$G_{n:i} = \frac{\Delta\theta_{n:i}}{\Delta\mathbf{d}_{n:i}} = \frac{\sqrt{\text{trace}\left(\mathbf{H}_{n:i}^{\mathrm{T}}\mathbf{H}_{n:i}\right)^{-1}}}{\sigma}, \tag{12}$$

where σ is the error variance in the environment and $\mathbf{H}_{n:i}$ denotes the jacobian matrix

$$\mathbf{H}_{n:i} = \begin{bmatrix}
\frac{\boldsymbol{\theta}_{n:x} - \boldsymbol{\varphi}_{1:x}}{d_{n1}} & \frac{\boldsymbol{\theta}_{n:y} - \boldsymbol{\varphi}_{1:y}}{d_{n1}} & 1 \\
\frac{\boldsymbol{\theta}_{n:x} - \boldsymbol{\varphi}_{2:x}}{d_{n2}} & \frac{\boldsymbol{\theta}_{n:y} - \boldsymbol{\varphi}_{2:y}}{d_{n2}} & 1 \\
\vdots & \vdots & \vdots \\
\frac{\boldsymbol{\theta}_{n:x} - \boldsymbol{\varphi}_{M_i^-:x}}{d_{nM_i^-}} & \frac{\boldsymbol{\theta}_{n:y} - \boldsymbol{\varphi}_{M_i^-:y}}{d_{nM_i^-}} & 1
\end{bmatrix}. \tag{13}$$

From Eq. (12), it is seen that an error variance σ is required to calculate the GDOP. However, in practice, each target-to-anchor link will have different error variances. Therein, the GDOP minimum criterion is not sufficient in representing an accurate target location due to different error variances. As such, using an estimated set of distances $\hat{d}_{nm} = \left\|\boldsymbol{\varphi}_m - \hat{\boldsymbol{\theta}}_{n:i}\right\|$ between the initial estimates and anchors, we introduce a Weighted GDOP (w-GDOP) using a weighted matrix $\mathbf{W}_{n:i}$ to replace the error variance σ

$$\mathbf{W}_{n:i} = \begin{bmatrix}
w_{n1} & 0 & \cdots & 0 \\
0 & w_{n2} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & w_{nM_i^-}
\end{bmatrix}, \tag{14}$$

where the main diagonals are $w_{nm} = \left(1 - \frac{\tilde{d}_{nm} - \hat{d}_{nm}}{\text{Median}(\tilde{\mathbf{d}}_{n:i})}\right)^2$, Median$(\tilde{\mathbf{d}}_{n:i})$ is the median of the measured distances $\tilde{\mathbf{d}}_{n:i}$.

In Eq. (13), the observation matrix $\mathbf{H}_{n:i}$ is seen to be dependent on the true distances $\mathbf{d}_{n:i}$ and the true target location $\boldsymbol{\theta}_{n:i}$

which are unknown. Therefore, these are replaced with the set of measured distances $\tilde{\mathbf{d}}_{n:i}$ and the initial target estimate $\hat{\boldsymbol{\theta}}_{n:i}$ which is obtained from our weighted min-max algorithm. The jacobian matrix $\mathbf{H}_{n:i}$ is modified by removing the vector $\mathbf{1}_{M_i^-}$ column to improve its accuracy. The objective of the weight matrix $\mathbf{W}_{n:i}$ is to present a simple strategy to weight the imperfect measured distances $\tilde{\mathbf{d}}_n$ so as to significantly reduce the noise and bias in measured distances and target estimates. Therefore, the standard GDOP in Eq. (12) is modified and a weighted GDOP is presented as

$$\mathcal{G}_{n:i} = M_i^- \sqrt{\text{trace}\Big(\mathbf{H}_{n:i}^{\mathrm{T}} \mathbf{W}_{n:i}^{-1} \mathbf{H}_{n:i}\Big)^{-1}}, \tag{15}$$

B. Location Estimation Based on the weighted GDOP

To evaluate the different anchor clusters, an initial location estimate $\hat{\boldsymbol{\theta}}_{n:i}$ is calculated for each cluster $C_i(\boldsymbol{\Phi}^-)$ utilizing the weighted min-max algorithm in Eq. (11). The weighted GDOP $\mathcal{G}_{n:i}$ in Eq. (15) for each cluster is used as an indicator of the estimation accuracy. Then, a simple way to obtain a final estimate is to select the initial estimate $\hat{\boldsymbol{\theta}}_{n:i}$ of the anchor cluster with the minimum $\mathcal{G}_{n:i}$ for target localization, called **minimum w-GDOP** algorithm.

Next, we present a **combined w-GDOP** algorithm that estimates a final location using the set of initial estimates and their w-GDOP values as follows:

$$\hat{\boldsymbol{\theta}}_n = \sum_{i=1}^{c} \frac{\hat{\boldsymbol{\theta}}_{n:i}}{\mathcal{G}_{n:i}} \Big/ \sum_{i=1}^{c} \frac{1}{\mathcal{G}_{n:i}}. \tag{16}$$

We use the inverse of the w-GDOP values as a weight for each initial estimate such that higher GDOP values are penalized. The pseudocode for target localization based on the minimum and combined w-GDOP is shown in Algorithm 1.

VI. Performance Evaluation

Considering the importance of anchor selection in target localization, it becomes crucial that the proposed anchor cluster based on strong RSS measurements, weighted min-max algorithm and weighted GDOP techniques are analyzed using simulations and real-time experiments.

Algorithm 1 - Target Localization using Weighted GDOP

1: **Input**: Anchors $\boldsymbol{\Phi} = \big[\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \cdots, \boldsymbol{\varphi}_M\big]$ and Measured Distance Matrix $\tilde{\mathcal{D}} = \big[\tilde{\mathbf{d}}_1, \tilde{\mathbf{d}}_2, \cdots, \tilde{\mathbf{d}}_N\big]^{\mathrm{T}}$ (given)
2: **Iteration**: To estimate $\hat{\boldsymbol{\theta}}_n \;\forall\; n$.
3: **Anchor Clusters**: Different anchor clusters $C_i(\boldsymbol{\Phi}^-)$, $i = 1, \cdots, c$ are created using the signal strength and variance of the received signals between all anchors and target.
4: **Representation**: Measured distances $\tilde{\mathbf{d}}_n^- = \big\{\tilde{d}_{n1}, \tilde{d}_{n2}, \cdots, \tilde{d}_{nM_i^-}\big\}$ corresponding to the index i is represented as $C_i(\boldsymbol{\Phi}^-) = \Big\{\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \cdots, \boldsymbol{\varphi}_{M_i^-}\Big\}$.
5: **Initial Location Estimate**: The weighted min-max algorithm in (11) is used to obtain the target estimate $\hat{\boldsymbol{\theta}}_{n:i}$
6: **GDOP Values**: The weighted GDOP $\mathcal{G}_{n:i}$ is computed for each cluster $i = 1, 2, \cdots, c$.
7: **Final Location Estimate**: Final estimate $\hat{\boldsymbol{\theta}}_n$ is obtained by either selecting the initial estimate of the anchor cluster with the minimum w-GDOP or combining all anchor clusters using Eq. (16).
8: **Output**: $\hat{\boldsymbol{\theta}}_n \;\forall\; n$.

A. Simulation Analysis

Here, two simulation analyses are performed for the evaluation of the proposed anchor selection and localization algorithms and existing techniques. First, the proposed anchor selection techniques and the weighted min-max algorithm are evaluated. In the second simulation, our proposed technique is compared against the existing techniques in [7], [9] and [10].

1) Proposed Technique: Here, we consider a large indoor area of $60m \times 60m$ as seen in the localization topology in Figure 2 with obstacles to depict NLOS and multipath conditions. For the network, $M = 16$ anchors were placed using tight frames and 196 uniformly spaced targets placed in the area $4m$ apart. The RSS $\boldsymbol{\varphi}_{\tilde{r}_{nm}}$ between target n and anchor m to obtain the pairwise distance between them are modelled using the log-normal shadow model with path loss exponent and standard deviation parameters $(\alpha = 3, \sigma = 8)$ to obtain the target location. The parameters (α, σ) for the wireless channel are chosen based on the results in [4].

For evaluation, multiple anchor clusters are needed to estimate an initial target's location. Therefore, to utilize any localization algorithm, it becomes necessary to determine the optimal number of clusters c needed for target localization. As such, the average root mean squared error of the target points $\boldsymbol{\Theta}$ are plotted against $M_i^- = 3, 4, 5, 6, 7, 8$ anchors with the strongest RSS $\boldsymbol{\varphi}_{\tilde{r}_{nm}}$. The MSEs are obtained using the min-max and weighted min-max algorithms for target localization in Figure 3a. Clearly, it is seen that the weighted min-max outperforms the min-max algorithm. In the weighted min-max, $M_i^- = 4$ significantly reduces the RMSE by 43% in comparison to using all $M = 16$ anchors, closely followed by $M_i^- = 5, 6$. The results of $M_i^- = 4, 5, 6$ anchors with the strongest RSSs $\boldsymbol{\varphi}_{\tilde{r}_{nm}}$ are illustrated for all N targets. In Figure 3b, the RMSE of each anchor cluster $C_i(\boldsymbol{\Phi}^-)$, with $M_i^- = 4, 5, 6$ anchors are plotted against the RMSE of all $M = 16$ anchors for all target locations.

Next, the weighted GDOP is analysed by evaluating the set of three ($c = 3$) anchor clusters ($M_i^- = 4, 5, 6$ anchors with the strongest RSSs $\boldsymbol{\varphi}_{\tilde{r}_{nm}}$) and their initial estimates $\hat{\boldsymbol{\theta}}_{n:i}$ so as to obtain accurate estimations for all targets. Here, the final estimates of the target location is computed by selecting the initial estimate $\hat{\boldsymbol{\theta}}_{n:i}$ of the anchor cluster with the minimum $\mathcal{G}_{n:i}$ or combining the initial estimates $\hat{\boldsymbol{\theta}}_{n:i}$ anchor clusters using an inverse average $1/\mathcal{G}_{n:i}$ in Eq. (16). The results of the proposed w-GDOP techniques are obtained for analysis. In Figure 4, the RMSE of the final target estimates

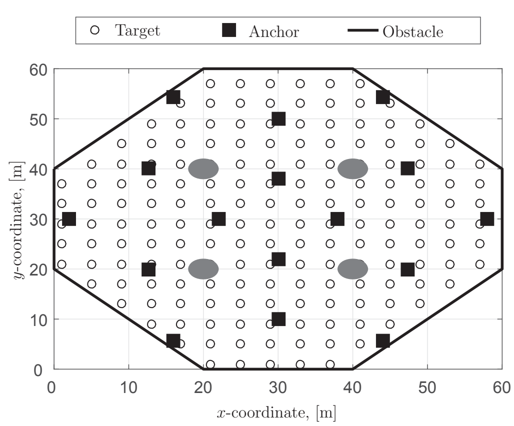
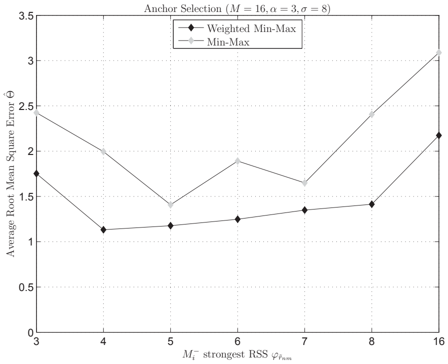
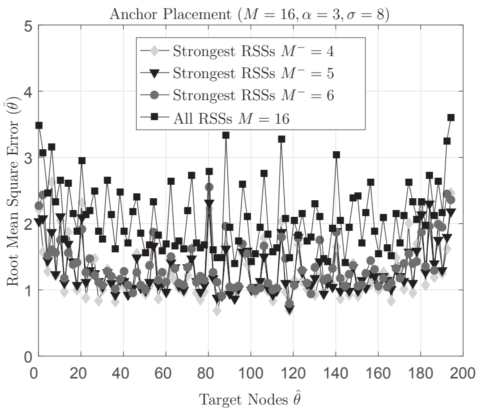


Figure 2: Localization topology for $M = 16$ anchors and $N = 196$ targets.



(a) Strongest signals techniques



(b) All signals and strongest signals techniques

Figure 3: Target localization using anchor clusters based on strongest RSSs

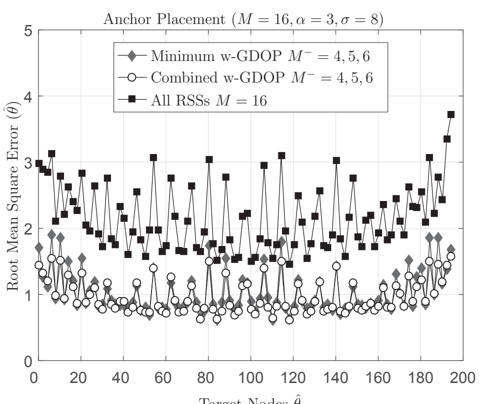
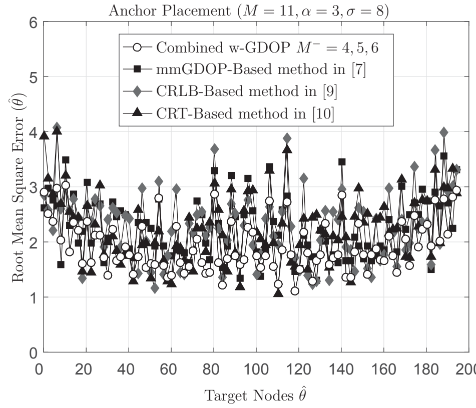


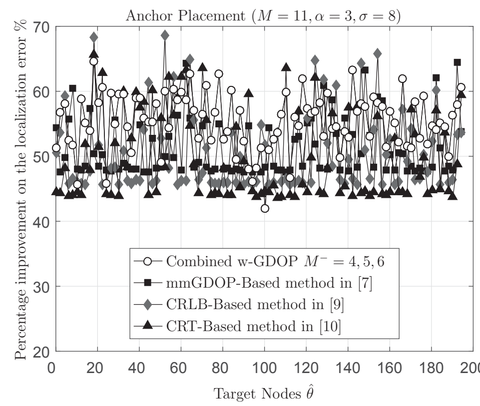
Figure 4: Target localization using w-GDOP to evaluate anchor clusters.

of the minimum w-GDOP and combined w-GDOP are plotted against the RMSE of all $M = 16$ anchors at different target locations.

2) Comparison with Existing Techniques: Here, we analyse and compare the proposed anchor selection technique and the weighted min-max localization algorithm with the existing anchor selection and localization algorithms using the same topology as in the previous subsection. Therefore, for each target point, the objective is to perform target localization by 1) combining the w-GDOP $\mathcal{G}_{n:i}$ and initial estimates $\hat{\boldsymbol{\theta}}_{n:i}$ of multiple anchor clusters $C_i(\boldsymbol{\Phi}^-)$ obtained by selecting $M^- = 4, 5, 6$ anchors with the strongest RSSs and the weighted min-max to obtain a final estimate $\hat{\boldsymbol{\theta}}_n$; 2) selecting anchor clusters based on the GDOP using matrix multiplication (mmGDOP) and localization algorithm in [7]; 3) selecting anchor clusters based on the Cramér-Rao lower bound (CRLB) and the localization algorithm in [9] and 4) selecting anchor



(a) Proposed and existing techniques



(b) Proposed and existing techniques

Figure 5: Target localization using anchor selection techniques.

clusters based on the classification and regression tree (CRT) localization algorithm in [10].

The results of these anchor selection techniques for target localization are shown in Figure 5. In Figure 5a, the RMSE of the combined w-GDOP and existing techniques in [7], [9], [10] were plotted against one another for all target locations. It is seen that the RMSE on target location estimate can be further reduced using each of the above anchor selection techniques for target localization. Clearly, combining the w-GDOP of multiple anchor clusters ($M^- = 4, 5, 6$) outperforms the anchor selection techniques in [7], [9], [10] for most of the target test points. Furthermore, Figure 5b is shown to illustrate the percentage reduction in the RMSE of target estimates achieved using anchor selection techniques. The percentage improvement on the localization estimation errors obtained utilizing all $M = 11$ anchors varies between $[40\% \to 60\%]$. **From the above results, it is seen that the proposed anchor cluster selection technique based on combining the weighted GDOP of anchor clusters improves performance with an average of 57% and leads to a more accurate target localization than existing methods with an average of 43% for [7], 47% for [9] and 50% for [10].**

B. Experimental Analysis

In sub-section VI-A2, our proposed anchor selection technique was compared against the existing anchor selection techniques and localization algorithms in [7], [9] and [10] using a cross topology. For better and concrete performance evaluation, the proposed and existing techniques were evaluated by performing real-world experiments at the 2017 Hong

Figure 6: Target Localization for $M = 12$ anchors and $N = 12$ targets.

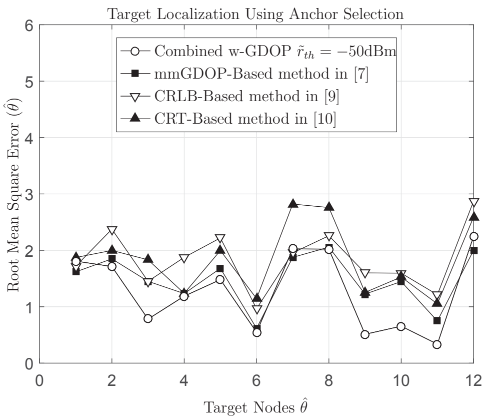


Figure 7: Target localization using real-world experiments.

Kong Watch and Clock Fair in the presence of humans and obstacles to provide strong multipath and NLOS conditions.

RSS measurements were obtained from WiFi access points acting as anchors and a mobile phone acting as a target. To this end, anchors and target test points were deployed in a large indoor $60m \times 60m$ exhibition centre with about 100 cubicles leading to high NLOS and multipath propagations. In the exhibition centre, $M = 12$ anchors and $N = 12$ targets were placed at coordinates as seen in Figure 6. The anchors were placed $2m$ above the cubicles with the targets handheld and RSSs were taken for time window $\mu = 4$mins at each target points. Due to the relationship between the distance and RSS in Eq. (6), the received signal strength $P_r(d_{nm})$ between anchor $\boldsymbol{\varphi}_m$ and target $\boldsymbol{\theta}_n$ are converted into measured distance $\tilde{d}_{nm}$. A transmitted signal power of $P_{t:n} = 10$dBm is set a priori for all targets $\boldsymbol{\Theta}$ and the path loss exponent $\alpha = 2.87$ was calculated using Minimum MSE.

For each target point, the objective is to perform target localization by 1) combining the w-GDOP $\mathcal{G}_{n:i}$ and initial estimates $\hat{\boldsymbol{\theta}}_{n:i}$ of multiple anchor clusters $C_i(\boldsymbol{\Phi}^-)$ obtained by predefining a RSS threshold $\tilde{r}_{th} = -50$dBm to screen out weak RSSs (some anchors may be out of coverage) and the weighted min-max to obtain a final estimate $\hat{\boldsymbol{\theta}}_n$; 2) selecting anchor clusters based on the GDOP using matrix multiplication (mmGDOP) and localization algorithm in [7]; 3) selecting anchor clusters based on the CRLB and the cooperative localization algorithm in [9] and 4) selecting anchor clusters based on the classification and regression tree (CRT) localization algorithm in [10].

The results of these anchor selection techniques for target localization are shown in Figure 7. It is seen that the RMSE on target location estimate can be reduced using effective anchor selection techniques for target localization. Clearly, combining the w-GDOP of multiple anchor clusters outperforms the existing anchor selection techniques in [7], [9], [10]. From the above results, it is clear that the proposed anchor cluster selection technique based on combining the weighted GDOP of anchor clusters improves performance and leads to the minimum RMSE of target estimates in comparison to others.

VII. Conclusions

In this paper, efficient localization techniques for anchor clusters selection were presented for target localization to significantly minimize the RMSE on the target's location estimate. A simple and accurate weighted min-max algorithm for estimating the location of targets is also presented. Different anchor clusters lead to different RMSE at different target locations and indoor conditions. As a result, an accurate selection of the anchors cluster corresponding to the minimum RMSE or a resultant location based on the weights of the weighted GDOP without a priori knowledge of the target's true location was required. Therefore, a weighted GDOP to efficiently evaluate anchor clusters was proposed for accurate and improved target localization.

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